Investigation of Vertical and Horizontal Momentum Transfer in the Gulf of Mexico Using Empirical Mode Decomposition Method

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(Manuscript received 27 October 2003, in final form 9 December 2004)

ABSTRACT

Data from a series of deep mooring stations in the Gulf of Mexico (GOM) have been analyzed with the newly developed empirical mode decomposition and Hilbert spectral analysis method, abbreviated as Hilbert–Huang transformation (HHT). The flows in the GOM near the shelf/slope region are treated as a two-layer system, with the 800-m permanent thermocline as the dividing depth. When the data are treated with HHT, motions of different temporal scales are identified. The top layer (depth less than 800 m) is controlled by inertia flow with episodic Loop Current eddies, while the lower layer (depth greater than 800 m) is controlled primarily by the topographic Rossby waves and small-scale cyclonic and anticyclonic eddies. Using a cross-correlation analysis between the appropriate intrinsic mode components from the data, the wavelength, the phase velocity, and the vertical trapping depth for the topographic Rossby waves were determined. Observations are in general agreement with the modeled results by Oey and Lee.

1. Introduction

The Gulf of Mexico (GOM) is a semienclosed sea. The strong and warm Loop Current (LC) enters from the Caribbean Sea through Yucatan Strait and occasionally sheds Loop Current eddies (LCEs) before it moves out to form the Gulf Stream in the Atlantic Ocean. The LC is a dominate feature in the GOM, but the variations of the wind forcing by cyclogenesis, tropical storms, and the river input from the mighty Mississippi River all provide additional energy into the system. With these additional forcings, the dynamics of the gulf become extremely complicated. Though the upper-ocean dynamics of the GOM have been studied extensively, primarily through field measurements, modeling efforts, and remote sensing techniques (Hamilton et al. 2000; Sturges and Leben 2000; Wang et al. 2003), the circulation patterns within the water column in the deep water region have, so far, mostly been overlooked (Nowlin et al. 2001). Phenomena such as the diversion of water mass from the LC, vertical momentum transfer by storms, and the strong bottom currents associated with topographic-trapped waves are all important components for a detailed description of dynamic processes in this region. In this paper, we will try to fill in some of the knowledge gaps by using the data from the pilot investigation related to the shelf/slope dynamics studies sponsored by the Minerals Management Service (MMS).

The development of deep water oil exploration and extraction in the GOM region has increased rapidly in recent years. Incidences of strong bottom currents have been reported routinely by offshore operators during their deep water exploratory operations; yet little data were available for us to delineate their dynamics. As a result, a series of deep mooring stations designed by the MMS (McKay et al. 2001) have been developed to study the shelf/slope/rise dynamics in order to fill the information gap. One of the pilot studies for deep water currents was completed last year. The data collected include bottom pressure, velocity, temperature, and salinity depth profiles from various current meters and other sensors. Past examinations of the data were lim-
ited to the traditional spectral analysis. In the traditional approach, however, the nonstationary and nonlinear effects are usually glossed over, and the results are concluded only on the determination of the temporal scales of the motions in a Fourier-based analysis. Here we use a new technique, the empirical mode decomposition (EMD) method developed by Huang et al. (1996, 1998, 1999, 2003) and Wu and Huang (2004) to study the data with special attention to the nonstationarity of the processes. Our goal is to examine the dynamics of the sporadic strong bottom currents and characterize the variation of the flow patterns and the energy sources of these deep currents throughout the water column. By using the intrinsic mode components, we have determined that the flows in the water column consisted of three types: the strong inertial flow at the surface, the LCE in the upper layer, and the TRW (topographic Rossby waves) confined in the lower layer below the main thermocline, while cyclonic and anticyclonic eddies cross over the whole water column. Furthermore, we have also used a statistical method to discover the propagation of the motion and determine the wavelength, phase velocity, and vertical trapping depth of the TRW observed.

The paper is divided into five parts. Following this short introduction, we will present first the data used in this study and then the new method used in our analysis. Because the method is relatively new, we will give a brief summary of the essence of the methodology. Next, we will present the salient results. Last, we present our discussions and conclusions.

2. Field data

In the study, one tall mooring and two short moorings were deployed in deep water (2000 m) areas of the central GOM for 2 years, covering a total of 740 days from August 1999 to August 2001. The locations of three stations are given in Fig. 1. The tall mooring (I1) is an extensively instrumented mooring that consists of four acoustic Doppler current profilers (ADCPs) to measure the velocity profile at intervals of either 4 or 8 m for the upper layer (800 m and less). The bottom layer was measured by conventional current meters at 200-m intervals. In addition, temperature and salinity profiles were also measured. Two short moorings, referred to as I2 and I3 (Hamilton et al. 2003), each consists of three current meters near the bottom at 200-m intervals. The lowest current meter is within 20 m from the bottom.

All data records underwent basic quality assurance during which suspect data values were flagged. Short gaps of a few hours caused by flagged data were filled by linear interpolation. The records at each depth level were merged into a continuous time series. The original data were recorded at 30–60-min intervals, filtered with a 3-h, low-pass Lanczos kernel to minimize the noise, and decimated to 1-h intervals. The axes of the current velocity records were rotated so that the U component (east–west) is directed along the general trend of the isobaths, and the V component (north–south) is directed across the isobath position (see also Hamilton et al. 2003).

In this study, we have concentrated on the velocity data, using the current data from the tall mooring to examine the velocity variation in the water column and from the two short moorings to examine the spatial variation. Our attention is primarily on the deep water movements.

3. Analysis method

Most of the existing data analysis methods are designed for linear and stationary processes. The traditional Fourier spectral analysis and the various probability distributions are cases in point. Yet, natural phenomena are seldom stationary, and many ocean
dynamic processes are certainly nonlinear. Facing data from such a complicated natural environment as exists in the GOM, most investigators cavalierly ignore the limitations of the methods. Here we decided to try a new method, the EMD method and the Hilbert spectral analysis (Huang et al. 1996, 1998, 1999, 2003; Wu and Huang 2004), designed especially for nonstationary and nonlinear processes. Since the method is new, a brief summary is given below.

The key part of the method is the EMD, which can decompose any complicated data into a finite and often small number of intrinsic mode functions (IMFs), while an IMF is defined as any function having the same number of zero-crossings and extrema, and also having symmetric envelopes defined by the local maxima and minima, respectively. The IMF admits well-behaved Hilbert transforms, as discussed by Huang et al. (1998, 1999). This decomposition method is adaptive and highly efficient. Since the decomposition is based on the local characteristic time scale of the data, it is applicable to nonlinear and nonstationary processes. With the Hilbert transform, the IMFs yield instantaneous frequencies as functions of time that give sharp identification of imbedded structures. The final presentation of the results is an energy–frequency–time distribution, designated as the Hilbert spectrum. The following is a brief summary based on Huang et al. (1998):

For an arbitrary \( L^p \) class time series, \( X(t) \), we can always compute its Hilbert transform, \( Y(t) \), as

\[
Y(t) = \frac{1}{\pi} P \int \frac{X(t')}{{t'-t}} \, dt',
\]

where \( P \) indicates the Cauchy principal value. With this definition, \( X(t) \) and \( Y(t) \) form a complex conjugate pair, so that we have an analytic signal, \( Z(t) \), as

\[
Z(t) = X(t) + iY(t) = a(t) e^{i \theta(t)},
\]

in which

\[
a(t) = \left[ X^2(t) + Y^2(t) \right]^{1/2},
\]

\[
\theta(t) = \arctan \frac{Y(t)}{X(t)}.
\]

Then, the instantaneous frequency is

\[
\omega(t) = \frac{d\theta(t)}{dt}.
\]

Contrary to the suggestion given by Hahn (1996), one should not take just any data, perform a Hilbert transform, find the phase function, and define the instantaneous frequency, as given in Eq. (4), which will give frequency, sometimes negative, that will not make physical sense. The real advantage of the Hilbert transform only became viable after Huang et al. (1996, 1998) introduced the EMD method with the following steps.

Identify all the local extrema, and then connect all the local maxima by a cubic spline line as the upper envelope. Repeat the procedure for the local minima to produce the lower envelope. The upper and lower envelopes should cover all the data between them. The mean of the upper and lower envelopes is then designated as \( m_1 \), and the difference between the data and \( m_1 \) is the first “proto-intrinsic mode function,” \( h_1 \), that is,

\[
X(t) - m_1 = h_1.
\]

This process should be applied repeatedly. In the subsequent sifting processes, \( h_1 \) is treated as the data, and then

\[
h_1 - m_{11} = h_{11},
\]

Repeated siftings, up to \( k \) times, yield

\[
h_{1;k-1} - m_{1k} = h_{1k};
\]

and while \( h_{1k} \) becomes an IMF. It is designated

\[
c_1 = h_{1k},
\]

as the first IMF component from the data.

Overall, \( c_1 \) should contain the finest scale or the shortest period component of the signal. We can separate \( c_1 \) from the rest of the data by

\[
X(t) - c_1 = r_1.
\]

Since the residue, \( r_1 \), still contains longer period components, it is treated as the new data and subjected to the same sifting process as described above. This procedure can be repeated to obtain all subsequent \( r_i \), and the result is

\[
r_1 - c_2 = r_2,
\]

\[
\vdots
\]

\[
r_{n-1} - c_n = r_n.
\]

This process will end finally when the residue, \( r_n \), becomes a constant, a monotonic function, or a function with only one maximum and one minimum from which no more IMF can be extracted. By summing Eqs. (9) and (10), we finally obtain

\[
X(t) = \sum_{j=1}^{n} c_j + r_n.
\]

Thus, we achieve a decomposition of the data into \( n \) empirical modes, and a residue, \( r_n \), which can be either the mean trend or a constant. EMD is a Reynolds-type decomposition. Although no orthogonality condition is imposed, the IMFs are nearly orthogonal to each other.
The nonorthogonal leakage for nonlinear data is natural for, when the processes are nonlinear, orthogonality among decomposed components would not necessary hold. Using IMF, we can devise a time–space filtering approach. For example, a low-pass filtered result of a signal having $n$th IMF components can be simply expressed as

$$X_{lk}(t) = \sum_{k}^{n} c_j + r_n,$$  \hspace{1cm} (12)

and the high-pass results can be expressed as

$$X_{hk}(t) = \sum_{k}^{n} c_j;$$  \hspace{1cm} (13)

further, a band-pass result can be expressed as

$$X_{bk}(t) = \sum_{b}^{k} c_j.$$  \hspace{1cm} (14)

The advantage of this time–space filtering is that the results preserve the full nonlinearity and nonstationarity in physical space.

Contrary to almost all other earlier methods, this new method is intuitive and direct; its basis is a posteriori and also adaptive for it is based on and derived from the data.

Having obtained the IMF components, one can apply the Hilbert transform to each and compute the instantaneous frequency according to Eq. (4). After performing the Hilbert transform on each IMF component, the original data become the real part (RP) of the following expansion:

$$X(t) = \text{RP} \sum_{j=1}^{n} a_j(t)e^{i(f_{ao}t)}dt.$$  \hspace{1cm} (15)

The signal given in Eq. (15), presented in time–energy–frequency space as $H(\omega, t)$, is designed as the Hilbert spectrum.

Equation (15) gives both amplitude and frequency of each component as functions of time. The same data, if expanded in a Fourier representation, would be

$$X(t) = \text{RP} \sum_{j=1}^{n} a_j e^{i\omega_j t},$$  \hspace{1cm} (16)

with both $a_j$ and $\omega_j$ as constants. The contrast between Eqs. (15) and (16) is very clear: The IMF represents a generalized Fourier expansion. The Hilbert spectrum is sharper than any Fourier spectrum (see, e.g., Huang et al. 1998). If amplitude squared is more preferred, then the squared values of amplitude can be substituted to produce the Hilbert energy spectrum just as well.

With the Hilbert Spectrum defined, we can also define the marginal spectrum, $H(\omega, t)$, as

$$h(\omega) = \int_{0}^{T} H(\omega, t) dt.$$  \hspace{1cm} (17)

The marginal spectrum offers a measure of the total amplitude (or energy) contribution from each frequency value. It represents the cumulated amplitude over the entire data span in a probabilistic sense. The combination of the EMD and the Hilbert spectral analysis is abbreviated as the Hilbert–Huang transform (HHT). More details can be found in Huang et al. (1996, 1998, 1999, 2003).

4. Data analysis and results

Having presented the data and the analysis method, we can now present the results. For this study, we will use the velocity data from mooring stations I1, I2, and I3. The east–west components from station I1 at depths of 154 and 1800 m are given in Figs. 2a and 2b. From Fig. 2a, we can immediately identify the certain episodic events as the possible passage of the LC and the LCEs (Eddy J and Eddy M) during August 1999–August 2001 simply from the variation of the current magnitudes during these 2 years. The signals from the LC and LCEs are not so clear in the current data at the 1800-m depth. Other than this casual observation, we will use the new method to probe the deeper underlying dynamics. We focus our efforts on the transfer mechanism of surface flows to the deep water flows from the field data. Data at four water depths (154, 800, 1000, and 1800 m) are selected for analysis in detail. The data represent the surface layer, transition layer, and bottom layer. The data were first processed by EMD and followed by marginal Hilbert spectrum for further investigation.

a. Empirical mode decomposition

The $U$ and $V$ currents at four selected elevations are analyzed by using the EMD method. The resulting IMF components at depths of 154 and 1800 m are given in Figs. 2c (for 154 m) and 2d (for 1800 m) as examples. The data at 154 m yield 13 IMF components, while the data at 1800 m yield 12. Either way, the number of the components is less than the $\log_2 N$, the theoretical limit of EMD working as a bank of a dyadic filter, as proposed by Flandrin et al. (2004) and Wu and Huang (2004). As one can see, the magnitudes of the components are not monotonically decreasing, indicating that the data are not white noise; therefore, the IMF indeed
contains information at certain scale ranges. The simple criterion proposed by Wu and Huang (2004) for judging the information contained is to examine the magnitude of the components: For example, the 10th component at the 154-m station is the most energetic. It is certainly significant, for it most likely represents the smoothed current of the two LCE J and M events, respectively. We will discuss this later.

One of the interesting features of the IMF is the last component, which represents the residue after all the oscillatory components are removed. At 154 m, the magnitude of the mean value is at 0.149 m s\(^{-1}\), representing a substantial mean eastward current at this depth. At 1800 m, the mean value of the last component is at \(-0.029\) m s\(^{-1}\), representing a small, but still significant, westward mean current. The comparison between the IMF’s residue from Eq. (11) with traditional arithmetic mean is shown in Fig. 3. The mean velocities \(U\) (data in circle) and \(V\) (data in square), as shown in this figure, from two methods all agree well. Two mean values from the 154-m station show some deviation. This reveals the meandering characteristics of the LC and LCEs at the 154-m station. In addition, these two sets of data together indicate the existence of a mean vertical shear current at this location.

At the tall I1 mooring, in Fig. 2c, the most energetic component seems to reside in the long-wave, low frequency flow region (C9–C10). The temporal scales of such events suggest that the IMF C10 may represent the two LCEs (Eddy J and Eddy M) passing through the mooring station. The sea surface height (SSH) anomaly maps from Ocean Topography Experiment/European Remote Sensing Satellite 2 (TOPEX/ERS-2) altimetric data on 23 November 1999 (day 87) and 30 April 2001 (day 611), shown in Figs. 4a and 4b, capture the passages of Eddies J and M. They clearly identify Eddies J and M on days 87 and 611 of C10 at the 154-m station of Fig. 2c, but not at the deep water 1800-m station of Fig. 2d. The data on Eddy M ended too soon to cover the complete eddy flow, for the magnitude of the IMF component had barely returned to the zero level after the energetic event. To confirm our visual observations from the figures, we have computed the root-mean-squared (rms) energy of each IMF component. The computed results are listed in Table 1. In the table, the maximum rms values are shown in boldface, and the second largest values are marked with asterisks. An interesting pattern emerges: First, at all depths except the 800-m station, the most energetic components are clustering closely, showing near unimodal distributions with only a weak secondary maximum. But at the 800-m depth, there is a distinct bimodal distribution. This is true for both the \(U\) and \(V\) components. Second, most energetic components for the surface flow are associated with long period motions, while the most energetic components for the deep flow are all at intermediate periods.

More detailed examination of the IMF components reveals the following: The dynamics are dominated by the east–west flow; therefore, we will discuss the \(U\) component of the 154-m depth first. The larger values are in the C10–12 components, which show the domination of the LC and LCE. The weak secondary maximum at the 154-m depth is the C7 component that coincides with C6 and C7 of the deeper meter. This implies the passing of the cyclonic and anticyclonic eddies generated by the local shear flows at the site. The region of the 800-m depth coincides with the location of the seasonal thermocline depth, which is considered a transition zone of two-layer systems. At this depth, the largest value in \(U\) velocity is in C6, with the second largest value in C9 at 800 m. Its time scale agrees closely with C6 and C7 of the deeper meters. However, the largest value in \(V\) velocity at 800 m is in C3, with the secondary value in C7. This reveals the transition characteristics of 800 m. At this depth, the dominated flows vary from the penetrated inertial flow generated by surface storms, the passing cyclonic/anticyclonic flows generated by the shear, and the penetration of the LCE. At the bottom of the 1800-m depth, the largest values are all clustered at IMFs C6 and C7. This implies that the flows are dominated by the TRW and small-scale cyclonic/anticyclonic flows. The exact sense of the cyclonic/anticyclonic rotation will have to be determined in conjunction with the satellite data.

The detailed statistics listed in Table 1 suggest that the flows in the water column can be divided into three different groups: The first group has its maximum current value at or near the surface, which has a frequency near the inertial period. For example, IMFs C2 and C3 satisfy this criterion, and we can conclude that they represent primary inertial oscillations. The second group has its maximum current value near the bottom layer, and with a much longer frequency from 0.05 to 0.1 cycles per day (cpd). The magnitude of current, in general, decreases as the distance from the bottom increases. These components, as represented by IMFs C6 and C7 (for 1000 and 1800 m), are considered to be the TRW, which we will discuss in more detail later. The third group that consists of IMFs larger than C7 is a long wave with much lower frequency. The strength of the current tends to decrease with depth. These components represent LC, LCEs, and other geostrophic flows. To check this classification scheme, we have to compute the frequency of each IMF component in detail, which should not be based on the traditional
Fourier analysis because the processes are clearly non-stationary. Here we will use the Hilbert spectral analysis.

b. Hilbert spectrum

The full and the marginal Hilbert spectra based on all the IMF components are computed according to Huang et al. (1998, 1999), which are employed to investigate the flow characteristics in more detail. To examine the results in detail, we have concentrated the resolution power of the spectra into two subranges: the inertia range centered around 1.0 cpd and the subinertia range of about 0.1 cpd.

Figure 5a shows the marginal Hilbert spectra of the first group of inertial flows in an east–west direction at four depths. In this figure, the computed peak frequency varies between 0.8 and 1.0 cpd. The variation of peak frequency may represent the characteristics of the intrinsic natures of inertial flows. This is consistent with the computed inertial period, 26.2 h, and inertial frequency of 0.92 cpd from $2\pi/\Omega_0$, where $\Omega_0$ is the Coriolis parameter (Hamilton et al. 2003). The inertial flows can be identified through $U-V-T$ plots as shown in Fig. 6a. In this figure, a group of five inertial oscillations with a clockwise rotation pass the measuring site of the 154-m station in 5.4 days. This series of inertial flows have an estimated frequency of 0.93 cpd that is almost equal to

![Figure 2](image-url)
the computed frequency of 0.92 cpd. However, the inertial flows can also be identified in deep water with a distorted shape, decreased energy level, and shifted frequency. The inertial oscillations at the 1800-m station are shown in Fig. 6b. The estimated frequency of inertial oscillation at the 1800-m station is 0.98 cpd. The spectral energy also decreases rapidly as the depth increases. Thus, these components indeed represent the inertial oscillations.

Next, we will examine the subinertia group representing the TRW and the LCE. Most of the contributions to this range come from the longer period components from C6 to C10 for all the depths. The marginal Hilbert spectra of this group are shown in Fig. 5b, which
demonstrates the existence of two-layer flows. For the low-frequency portion (i.e., <0.04 cpd) the spectral energy of the upper layer is much larger than that of the lower layer, suggesting a stronger surface drift current. However, in the midfrequency portion (0.07–0.08 cpd), the bottom-layer spectral energy density is larger than that of the upper layer and diminishing at the surface layer. This strongly suggests the existence of TRW in the data. By closely examining the spectra at the mid-range in Fig. 5b, we find two wave groups with distinguished characteristics. There is a weak group of waves with peak frequency around 0.055 cpd; the spectral energy at the surface and bottom are almost equal with some reduction at mid depth. This implies the existence of cyclonic and anticyclonic flows at the measuring site, and those flows have been identified using $U-V-T$ plot with partial success. The results are plotted in Figs. 7a and 7b for surface and bottom layers in which the sum of IMF terms, C6 and C7, are used. At the 154-m layer (Fig. 7a), a group of cyclonic flow followed by anticyclonic flow and another smaller cyclonic flow are identified visually. The frequencies of these flows are estimated from the mean Hilbert spectrum as 0.057, 0.1, and 0.097 cpd, respectively. The shapes of these flow systems are not symmetric which indicates the transient nature of the nonlinear dynamic and unstable condition of the flows. Similar flow patterns at the bottom layer (1800-m station) are shown in Fig. 7b. The frequencies of these flows are estimated again from the Hilbert spectrum as 0.051, 0.077, and 0.092 cpd, respectively. The first cyclonic flow has the largest velocity with frequencies of 0.057 and 0.051 cpd, that is, the group computed from the Hilbert spectrum. The detail analysis of cyclonic and anticyclonic flows will be presented in another paper. At the frequency band from 0.078 to 0.10 cpd, the spectral energy at the bottom is the highest and decreases as the depth decreases. A similar enhancement of bottom current was observed in the past by Hogg (2000) over the slope of the Grand Banks. The results of this analysis seem to indicate the passage of several groups of TRW during the measuring period. We will further inspect the Hilbert spectrum and statistical analysis to locate the passage of TRW and cyclonic flows from the data.

Now, let us turn our attention to the full Hilbert spectrum. The three-dimensional prospective view of the Hilbert spectra derived from two mid-IMF terms, C6 and C7, at the surface (154 m) and bottom (1800 m) layers at station I1 are shown in Figs. 8 and 9. By comparing these two figures, with the aid of a two-dimensional plot (Figs. 10a,b), we will locate passage of cyclonic and anticyclonic eddies and TRW in a time domain. Limited by the available data, we have to resort to some very simple criteria for flow classification. We believe that two criteria can be used to separate the passage of cyclonic flow and TRW: First, if the spectral energies have similar amplitudes from surface to bottom layers and the instantaneous frequencies are very close, we consider the motions to be of cyclonic flows. In Fig. 9, there are two regions that fit these criteria. They are located right after the passage of the LCE, around the 50th to 150th and 400th to 500th day zones (see also Fig. 7), respectively. Second, if the amplitude of spectral energy at the bottom layer is much greater than at the surface layer, we consider those motions as TRW. In Fig. 9, there is a possible event of TRW passage around the 220th–350th day period, for a high-energy density exists in the Hilbert spectrum at 1800 m but is absent in the surface layer (Fig. 8). Thus we may have witnessed a possible passage of TRW before the passage of Eddy M.

Granted that, in this region, we may have coexisting TRW and cyclonic flow. Therefore, we have to use the instantaneous frequency to distinguish TRW from cyclonic flow, for they have nonoverlapping frequency ranges. The TRW here had a frequency of 0.055–0.1 cpd while the cyclonic flow had a frequency of 0.03–0.05 cpd before the arrival of Eddy M, based on the analyzed data. Also at the period when Eddy J passed through, the surface layer showed a clear range of lower frequency (0.03–0.05 cpd) energy larger than in the deep layer that was dominated by a higher frequency range (0.05–0.1 cpd). Unfortunately, this frequency discriminator is not sensitive enough to separate the flow when Eddy M passed the region. As we can see, Eddy M is a much stronger eddy, and it could penetrate much
Fig. 4. Sea surface height anomaly maps from TOPEX/ERS-2 altimetric data (courtesy of R. Leben, CCAR, 2004, personal communication): (a) 23 Nov 1999, and (b) 30 Apr 2001. The measuring site II is located at 27°18’N, 89°47’W.
component is almost in the water column. As a result, wave–current interaction and other nonlinear effects can mark the distinctive characteristics of each separated motion. We are trying to examine this and other data further to find a more precise way to separate the eddies from TRW motions using statistical analysis.

c. Statistical analysis

Statistical analysis is designed with the primary goal of determining the propagation of dynamic characteristics of the currents and TRW through the correlation among different stations and of the same station at different depths. The justifications are as follow: A high correlation coefficient (>0.5) indicates that the different stations are indeed observing the same coherent motion. Based on the well-known Taylor’s assumption, the phase lag will give the propagation time provided the IMF components are orthogonal to each other. Indeed, the orthogonal condition is always satisfied as discussed by Huang et al. (1998). A specific check of orthogonality for the case studied is given in Table 1, which indicates the orthogonality condition for this case is indeed satisfied. With the known distance of the stations, the propagation time can be computed easily. The statistical results came primarily from the correlation coefficients as listed in Tables 2, 3a, and 3b. The results in these tables can be further divided into two classes. The first class consists of coefficients in Tables 2 and 3a–c, which summarize the correlation of the data at the same station but at different depths. The second class consists of coefficients in Tables 4 and 5, which summarize the horizontal correlation between different stations measured at 200 m from the bottom; the phase lag will give the propagation time.

Now let us discuss the vertical correlation first. The cross-correlation of currents at different depths in the early stage of passing Eddy M (day 396 to 667) are shown in Table 2. Based on the marginal Hilbert spectral analysis, we selected two IMF components, C6 and C7, to investigate the passage of TRW and cyclonic flows. In the table, we use “a” to represent data from a depth of 154 m, “b” from a depth of 800 m, “c” from a depth of 1000 m, and “d” from a depth of 1800 m. The cross-correlation coefficient of currents U, in the east–west direction between depths 1800 and 1000 m (Rdc), is 0.663 with a phase difference of −22 h. The coefficient reaching to 0.833 with a phase difference of 2 h suggests that the flow of U component is almost in phase between 1000 and 800 m (Rcb). At the surface layer, the coefficient is reduced, and the phase difference increases. From this analysis, we conclude that this vertically decaying motion with the distance from the bottom is a passage of TRW during that observing period.

Similar cross-correlation analysis for flows after the passage of Eddy J (day 83 to 375) is summarized in Table 3a. In this table, the coefficients for currents U at these four depths are all higher than 0.55, and the phase differences are also small. These results together with the satellite data seem to suggest that the cyclonic flows dominate during this period of observation. However, we find that there is a passage of TRW shown in Fig. 9. To further examine the dynamics, the data are further divided into two more specific subperiods (from day 83 to 208, and from day 208 to 375) according to the previous discussion of Fig. 9 on detailed analysis. There is a relative quiet period of surface flow after the passage of Eddy J. But the passage of TRW between day 208 and 375 is clearly shown in Fig. 9. The cross-correlation of the data between day 83 and 208 is summarized in Table 3b and the cross-correlation analyses of day 208...
to day 375 are summarized in Table 3c. The strong correlation coefficients and small phase differences at all depths shown in Table 3b suggest the penetration of cyclonic flows from surface to bottom during this period. The strong correlation at the bottom layer and the weak correlation between the bottom and surface layers, as shown in Table 3c, suggest the passage of TRW during this observation period. The vertical cross coefficient and phase relation are shown in Figs. 11a and 11b for cyclonic flow and TRW. The differences between these two flows can only be distinguished from the cross-correlation coefficient and phase relation using the proper IMF components in the surface layer, which is the subject of the next section.

d. Characteristics of TRW

To study the horizontal transfer of the water mass at the deep layer, the data at 200 m above the seafloor at

![Fig. 5. Marginal Hilbert spectra of two groups of IMF at station II: (a) inertial flows at in the east–west direction (IMF C2 and C3) and (b) subinertial and long waves in the east–west direction (IMF C6–C10).](image)
stations I2 and I3 were analyzed using the EMD method. One of the measured data and analyzed IMF components at the depth of 1800 m at station I2 are shown in Figs. 12a and 12b. The flow characteristics of the $U$ component are similar to the data at station I1, as shown in Figs. 2a and 2c. After the EMD analysis, the dominating components are again centered at the IMF C6 and C7 components. The cross-correlation analyses between stations I1, I2, and I3 at the 1800-m depth are computed and listed in Tables 4 and 5. In Table 4, the period of analysis covered from day 396 to day 667 for the passage of Eddy M. The coefficient of cross-correlation between stations I1 and I2 (Rd12) is 0.631 with a phase lag of $-27$ h. A good correlation also exists between I2 and I3. The same analysis was performed for day 83 to day 375 after the passage of Eddy J (results in Table 5). The high correlation of 0.663 with a phase lag of $-40$ h only exists between I1 and I2. The correlations decrease between I1 and I3, and between I2 and I3. This again implies that the flow during this time period consists of TRW and cyclonic flows that have different paths from other TRW.

The distances between I1 and I2, I1 and I3, and I2 and I3 are all set to be 23 km according to the deploy-

Fig. 6. Inertial oscillations derived from IMF C2–C3 components at (a) 154-m station and (b) 1800-m station.
ment plan, although they may deviate slightly from this value after deployment. Without further measurement, however, we have chosen this value for our computations. Based on the Table 3c, we identify that there is a group of TRW passing with a frequency of 0.075 cpd (Figs. 8 and 9). We found that this group of TRW has a phase lag of \( H11002 \) for the \( U \) velocity traveling from stations I1 to I2 (Table 5). The phase velocity, \( C_u \), is computed as

\[
C_u = \frac{\omega}{k} = \frac{\Delta z}{\Delta \alpha},
\]

where \( \omega \) is frequency, \( k \) is wavenumber in the east–west coordinate, and \( \alpha \) is phase lag. Here, we have \( \Delta z = 23 \) km, \( \Delta \alpha = 40 \) h, and \( \omega = 0.075 \) cpd. From the above equation, we calculate the phase speed of TRW as 13.8 km day\(^{-1}\), and the wavenumber is \( 3.415 \times 10^{-5} \) m\(^{-1}\). The wavelength \( \lambda \) is 184 km as computed from

\[
\lambda = \frac{2\pi}{k}.
\]

Similarly, we identify a second group of TRW passing the site as shown in Table 2. The frequency of this
group is 0.1 cpd with a phase lag of −27 h for $U$ velocity (Table 4). Using Eqs. (17) and (18), we compute wavelength and the phase speed in the $x$ coordinate for the second group of TRW as 204.7 km and 20.5 km day$^{-1}$, accordingly. However, these values will be modified once we have added the $y$ components.

Based on quasigeostrophic dynamics, the dispersion relation of TRW is given by two equations (Pedlosky 1979; Pickart 1995):

$$\mu = N(k^2 + l^2 + \beta k/\omega)^{1/2}/f$$ and

$$\mu \tanh(\mu h) = N^2/(\omega f)(kh_x - lh_y),$$

where $h$ is the water depth, $N$ is the constant Brunt–Väisälä frequency, $f$ is the Coriolis parameter using the $\beta$-plane assumption and where $\beta$ is $\partial f/\partial y$, $k$ is the wavenumber in east–west ($x$) coordinate and $l$ is the wavenumber in north–south ($y$) coordinate, and $1/\mu$ is the vertical trapping scale of the wave.

The field data were collected at the central GOM region where the topographic slope is large. We assume that the planetary beta is small relative to the local slope. Eq. (20) can be simplified as

$$\mu = N(k^2 + l^2)^{1/2}/f.$$ (21a)

We will compute the trapping vertical depth of TRW passing through the measuring site by using the above equation. The Brunt–Väisälä frequency $N$ is assumed constant with a value of $1.3 \times 10^{-3}$ s$^{-1}$ at the GOM region (Oey and Lee 2002).

We have identified two TRW groups passing the measuring sites. The first group, with a wave frequency of 0.075 cpd, was detected after the passage of Eddy J. The statistical analysis of this TRW group is shown in Table 5. A strong correlation between stations I1 and I2 exists in the east–west direction ($x$ coordinate), with a coefficient of 0.663, but not in north–south direction ($y$ coordinate). The value of wavenumber in the $x$ coordinate was computed as $3.415 \times 10^{-5}$ m$^{-1}$. Since there is negligible correlation existing in the $y$ coordinate, the vertical trapping depth for this group of TRW is found to be slightly high at 1502 m by using Eq. (21a), but is very close to the observed magnitude.

Fig. 8. The three-dimensional plot of Hilbert spectra derived from mid-IMF terms, C6 and C7, at 154-m depth at station I1.
The second group of TRW was identified prior to the passage of Eddy M, with a frequency of 0.1 cpd. The statistical analysis of this group is shown in Table 4. In this group of TRW, the correlation between stations I1 and I2 exists in both the x and y coordinates. The phase lags for the x and y coordinates are −27 and −32 h, respectively. In our computation, we select both coordinates having the same phase lag of −30 h. The wave numbers in the x and y coordinates, k and l, are equal to 3.42 × 10^{-5} m^{-1}. The overall wavenumber is 4.84 × 10^{-5} m^{-1}. The overall wavelength and phase velocity of this group become 130 km and 13 km day^{-1}. The values are different from those we computed previously using the U component in the x-coordinate (east–west) direction only. The overall trapping depth of this group of TRW becomes 1182 m, which puts it right near the permanent thermocline depth.

It is interesting to note that these two TRW groups, though similar, seem to have slightly different properties. The TRW associated with Eddy J and Eddy M have frequency values of 0.075 versus 0.1 cpd, wavelengths at 184 versus 130 km, phase velocities at 13.8 versus 13 km day^{-1}, and trapping depths at 1502 versus 1182 m, respectively. All these values compare well with the modeling results by Oey and Lee (2002), which gives the TRW, of comparable frequency, a wavelength range from 80 to 200 km, a phase velocity range around 14 km day^{-1}, and a trapping depth range from 893 to 1588 m.

The wavelengths of these two groups are different (184 versus 130 km). The first group of TRW moves primarily along the isobath (x coordinate), with a longer wavelength. The second group of TRW moves in the northeast direction (upslope). The wavelength of this group may be shortened when they propagate into the slope shelf. We found some support for this assumption from statistical analyses. According to Table 5, the correlation between stations I1 and I2 is good, but the correlations between stations I1 and I3, and I2 and I3 are poor. We may speculate that this wave train propagates along the isobath without refraction from the slope. The second group of TRW has a frequency of 0.1 cpd (10-day period), as shown in Table 4. This wave train has very high correlations between stations I1 and I2, II and I3, and I2 and I3. This group of TRW may propagate into the slope shelf and refract from the slope near station I2.

Although the measured TRW shows that the wave energy is highest at the bottom layer and decreases upward, as shown in Fig. 5b, the statistical analysis indicates that they have a phase lag at different depths.
For example, the phase at 1800 m lags the phase at 1000 m by $-22$ h (Table 2). However, the phase lag between 1000 m and 800 m is only 2 h. This implies that the columnar characteristic of the TRW exists only in some, but not all, water depths. A phase lag of $-42$ h exists between 1800 and 1000 m, and a small phase lag of $-3$ h exists between 1000 and 800 m, as shown in Table 3c for the other group of TRW. The data indicate that the water mass moved by the TRW is not columnar at the bottom but becomes columnar near the upper edge of trapping depth. This phase lag could well be the effects of finite eddy viscosity, while the theoretical model is for idealized inviscid fluid. Nevertheless, the TRW observed here is in general agreement with

![Fig. 10. The two-dimensional plot of Hilbert spectra derived from mid-IMF terms, C6 and C7, at (a) 154-m depth and (b) 1800-m depth at station I1.](image-url)
Table 2. Correlation coefficients and phase relation of measured velocity IMF components C6 + C7 across vertical planes during the passage of Eddy M (days 396–667) (where $a = 154$ m, $b = 800$ m, $c = 1000$ m, and $d = 1800$ m).

<table>
<thead>
<tr>
<th>Correlation coefficients/$U$</th>
<th>Phase lag/$U$ (h)</th>
<th>Correlation coefficients/$V$</th>
<th>Phase lag/$V$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{dc}$</td>
<td>0.663</td>
<td>−22</td>
<td>0.662</td>
</tr>
<tr>
<td>$R_{cb}$</td>
<td>0.833</td>
<td>+2</td>
<td>0.446</td>
</tr>
<tr>
<td>$R_{ba}$</td>
<td>0.418</td>
<td>+382</td>
<td>0.382</td>
</tr>
</tbody>
</table>

5. Summary and conclusions

Use of the EMD and Hilbert spectral analysis on the current meter data enables us to delineate the flow into three typical types: inertial oscillation, the TRW, and LC-related currents and eddies.

Our primary objective is to investigate the flows in the deep layer. We focus on IMF components C6 and C7 that dominate the bottom flows. We have used HHT, correlation analysis, and with the help of satellite data to analyse the current meter data. From the above analysis, we conclude that there are at least two groups of TRW and cyclonic flows passing through the measuring site. The salient results from our study are summarized as follows:

1) The inertial oscillations generated by storms and eddy shears decay with depth and shift to subinertial flows at the bottom.

2) The LC and LCEs dominate the upper-layer flows up to 800 m and decay rapidly in deeper water.

3) Two groups of TRW with frequencies of 0.075 and 0.1 cpd are identified at the bottom, and they move upward and decay at the surface.

4) Two groups of TRW have the same wave numbers and phase velocities but different wavelengths; the long wave moves along the isobath, and the short wave propagates into the slope shelf and is refracted. All these characteristics are in agreement with the modeled results by Oey and Lee (2002).

5) Based on satellite data and HHT analysis, a cyclonic flow with a frequency of 0.055 cpd is identified after the passage of LC Eddy J and penetrates to the bottom.

6) In statistical analysis, the lower frequency group (0.2–0.002 cpd) has a cross-correlation of 0.585 between 154 and 800 m, which decreases to 0.324 between 154 and 1000 m, and reduces to −0.158 between 154 and 1800 m.

7) The data are divided into several groups for further statistical analysis to identify the TRW and cyclonic flows and to support the characterization of TRW and cyclonic and/or anticyclonic flows in the deep water layer.

Table 3. Correlation coefficients and phase relation of measured velocity IMF components C6 + C7 across vertical planes (where $a = 154$ m, $b = 800$ m, $c = 1000$ m, and $d = 1800$ m).

<table>
<thead>
<tr>
<th>Correlation coefficients/$U$</th>
<th>Phase lag/$U$ (h)</th>
<th>Correlation coefficients/$V$</th>
<th>Phase lag/$V$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) After passage of Eddy J (days 83–375)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{1dc}$</td>
<td>0.705</td>
<td>−33</td>
<td>0.432</td>
</tr>
<tr>
<td>$R_{1cb}$</td>
<td>0.747</td>
<td>+3</td>
<td>0.538</td>
</tr>
<tr>
<td>$R_{1ba}$</td>
<td>0.55</td>
<td>−16</td>
<td>0.318</td>
</tr>
<tr>
<td>(b) Cyclonic eddy (R2) after passage of Eddy J (days 83–208)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{2dc}$</td>
<td>0.735</td>
<td>−27</td>
<td>0.642</td>
</tr>
<tr>
<td>$R_{2cb}$</td>
<td>0.793</td>
<td>+7</td>
<td>0.495</td>
</tr>
<tr>
<td>$R_{2ba}$</td>
<td>0.727</td>
<td>−17</td>
<td>0.393</td>
</tr>
<tr>
<td>(c) For TRW (R3) after passage of Eddy J (days 208–375)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{3dc}$</td>
<td>0.726</td>
<td>−41</td>
<td>0.33</td>
</tr>
<tr>
<td>$R_{3cb}$</td>
<td>0.676</td>
<td>−3</td>
<td>0.623</td>
</tr>
<tr>
<td>$R_{3ba}$</td>
<td>0.427</td>
<td>+534</td>
<td>−0.285</td>
</tr>
</tbody>
</table>

Table 4. Correlation coefficient of IMF C6 + C7 between I1, I2, and I3 at the bottom during passage of Eddy M (days 396–667).

<table>
<thead>
<tr>
<th>Correlation coefficients/$U$</th>
<th>Phase lag/$U$ (h)</th>
<th>Correlation coefficients/$V$</th>
<th>Phase lag/$V$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{di2}$</td>
<td>0.631</td>
<td>−27</td>
<td>0.417</td>
</tr>
<tr>
<td>$R_{di3}$</td>
<td>0.519</td>
<td>−79</td>
<td>0.292</td>
</tr>
<tr>
<td>$R_{di23}$</td>
<td>0.545</td>
<td>−56</td>
<td>0.394</td>
</tr>
</tbody>
</table>

Table 5. Correlation coefficient of IMF C6 + C7 between I1, I2, and I3 at the bottom after passage of Eddy J (days 83–375).

<table>
<thead>
<tr>
<th>Correlation coefficients/$U$</th>
<th>Phase lag/$U$ (h)</th>
<th>Correlation coefficients/$V$</th>
<th>Phase lag/$V$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RR_{di2}$</td>
<td>0.663</td>
<td>−40</td>
<td>0.227</td>
</tr>
<tr>
<td>$RR_{di3}$</td>
<td>0.356</td>
<td>−103</td>
<td>−0.242</td>
</tr>
<tr>
<td>$RR_{di23}$</td>
<td>0.373</td>
<td>−23</td>
<td>−0.337</td>
</tr>
</tbody>
</table>
These momentum transfers suggest that the GOM flow can be modeled as a two-layer flow. The upper layer is dominated by the LC, LCEs, and inertial oscillation, and the bottom layer is dominated by TRW and some cyclonic and anticyclonic flows that exist in the whole water column. The sources of energy at the bottom need to be evaluated using other sources of data that cover enough of the spatial domain.

Fig. 11. Vertical cross coefficient and phase relation at different depths with a shift of vertical axis, in which “a” refers to 154 m, “b” refers to 800 m, “c” refers to 1000 m, and “d” refers to 1800 m: (a) cyclonic flows (r2) and (b) TRW (r3).
Acknowledgments. This research was supported by MMS internal research funds. The encouragements from Dr. James Kendall and editorial assistance provided by Ms. Eileen Lear are gratefully acknowledged. NEH was supported in part by NASA Oceanic Processes Program and an ONR Grant N0001403IP20094. We also express our deep appreciation to Dr. Peter Hamilton, Jim Singer, and Van Waddell, all of SAIC, for their collection efforts and preliminary analysis of the data, without which this study would be impossible.

REFERENCES
