Ensemble Empirical Mode Decomposition:
A Noise Assisted Data Analysis Method

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Abstract

A new Ensemble Empirical Mode Decomposition (EEMD) is presented. This new approach consists of sifting an ensemble of white noise-added signal and treats the mean as the final true result. Finite, not infinitesimal, amplitude white noise is necessary to force the ensemble to exhaust all possible solutions in the sifting process, thus making the different scale signals to collate in the proper intrinsic mode functions (IMF) dictated by the dyadic filter banks. As the EMD is a time space analysis method, the white noise is averaged out with sufficient number of trials; the only persistent part survives the averaging process is the signal, which is then treated as the true and more physical meaningful answer. The effect of the added white noise is to provide a uniform reference frame in the time-frequency space; therefore, the added noise collates the portion of the signal of comparable scale in one IMF. With this ensemble mean, one can separate scales naturally without any *a priori* subjective criterion selection as in the intermittence test for the original EMD algorithm. This new approach utilizes the full advantage of the statistical characteristics of white noise to perturb the signal in its true solution neighborhood, and to cancel itself out after serving its purpose; therefore, it represents a substantial improvement over the original EMD and is a truly noise assisted data analysis (NADA) method.

1. Introduction

Empirical Mode Decomposition (EMD) has been proposed recently (Huang, et al. 1998, Huang et al. 1999) as an adaptive time-frequency data analysis method. It has proven to be quite versatile in a broad range of applications for extracting signals from data generated in noisy nonlinear and non-stationary processes (see, for example, Huang and Shen, 2005, Huang and Attoh-Okine, 2005). As useful as EMD proved to be, it still leaves some difficulties unresolved.

One of the major drawbacks of the original EMD is the frequent appearance of mode mixing, which is defined as a single Intrinsic Mode Function (IMF) either consisting of signals of widely disparate scales, or a signal of a similar scale residing in different IMF components. Mode mixing is a consequence of signal intermittency. As discussed by Huang et al. (1998 and 1999), the intermittence could not only cause serious aliasing in the time-frequency distribution, but also make the individual IMF devoid of physical meaning. To alleviate this from occurring, Huang et al. (1999) proposed the intermittence test, which can indeed ameliorate some of the difficulties. However, the approach itself has its own problems: First, the intermittence test is based on a subjectively selected scale. With this subjective intervention, the EMD ceases to be totally adaptive. Secondly, the subjective selection of scales works if there are clearly separable and definable time scales in the data. In case the scales are not clearly separable but mixed over a range continuously, as in the majority of natural or man-made signals, the intermittence test algorithm with a subjectively defined time scales often does not work very well.

To overcome the scale separation problem without introducing a subjective intermittence test, a new noise-assisted data analysis (NADA) method is proposed, the Ensemble EMD (EEMD), which defines the true IMF components as the mean of an ensemble of trials, each consisting of the signal plus a white noise of finite amplitude. With this ensemble approach, we can clearly separate the scale naturally without any *a priori* subjective criterion selection. This new approach is based on the insight gleaned from recent studies of the statistical properties of white noise (Flandrin, et al., 2004, and Wu and Huang, 2004), which showed that the EMD is effectively an adaptive dyadic filter bank[1] when applied to white noise. More critically, the new approach is inspired by the noise added analyses initiated by Flandrin et al. (2005) and Gledhill (2003). Their results demonstrated that noise could help data analysis in the EMD.

The principle of the EEMD is simple: the added white noise would populate the whole time-frequency space uniformly with the constituting components of different scales separated by the filter bank. When signal is added to this uniformly distributed white background, the bits of signal of different scales are automatically projected onto proper scales of reference established by the white noise in the background. Of course, each individual trial may produce very noisy results, for each of the noise-added decompositions consists of the signal and the added white noise. Since the noise in each trial is different in separate trials, it is canceled out in the ensemble mean of enough trails. The ensemble mean is treated as the true answer, for, in the end, the only persistent part is the signal as more and more trials are added in the ensemble.

[1] A dyadic filter bank is a collection of band pass filters that have a constant band pass shape (e.g., a Gaussian distribution) but with neighboring filters covering half or double of the frequency range of any single filter in the bank. The frequency ranges of the filters can be overlapped. For example, a simple dyadic filter bank can include filters covering frequency windows such as 50 to 120 Hz, 100 to 240 Hz, 200 to 480 Hz, and et al.

The critical concept advanced here is based on the following observations:

1. A collection of white noise cancels each other out in a time space ensemble mean; therefore, only the signal can survive and persist in the final noise-added signal ensemble mean.
2. Finite, not infinitesimal, amplitude white noise is necessary to force the ensemble to exhaust all possible solutions; the finite magnitude noise makes the different scale signals reside in the corresponding IMF, dictated by the dyadic filter banks, and render the resulting ensemble mean more meaningful.
3. The true and physically meaningful answer of the EMD is not the one without noise; it is designated to be the ensemble mean of a large number of trials consisting of the noise-added signal.

This EEMD proposed here has utilized all these important statistical characteristics of noise. We will show that *the EEMD utilizes the scale separation principle of the EMD, and enables the EMD method to be a truly dyadic filter bank for any data. By adding finite noise, the EEMD eliminates mode mixing in all cases automatically. Therefore, the EEMD represents a major improvement of the EMD method*.

In the following, a systematic exploration of the relation between noise and signal in data will be presented. Studies of Flandrin et al. (2004) and Wu and Huang (2004) have revealed that the EMD serves as a dyadic filter for various types of noise. This implies that a signal of a similar scale in a noisy data set could possibly be contained in one IMF component. It will be shown that adding noise with finite rather than infinitesimal amplitude to data indeed creates such a noisy data set; therefore, the added noise, having filled all the scale space uniformly, can help to eliminate the annoying mode mixing problem first noticed in Huang et al. (1999). Based on these results, we will propose formally the concepts of noise-assisted data analysis (NADA) and noise-assisted signal extraction (NASE), and will develop a method called the Ensemble Empirical Mode Decomposition, which is based on the original Empirical Mode Decomposition method, to make NADA and NASE possible.

The paper is arranged as follows. Section 2 will summarize previous attempts of using noise as a tool in data analysis. Section 3 will introduce the EEMD method, illustrate more details of the drawbacks associated with mode mixing, present concepts of noise-assisted data analysis and of noise-assisted signal extraction, and introduce the EEMD in detail. Section 4 will display the usefulness and capability of the EEMD through examples. A summary and discussion will be presented in the final section.

2. A brief survey of Noise Assisted Data Analysis

The word “noise” can be traced etymologically back to its Latin root of “nausea”, meaning “seasickness.” Only in Middle English and Old French does it start to gain the meaning of “noisy strife and quarrel”, indicating something not at all desirable. Today, the definition of noise varies in different circumstances. In science and engineering, noise is defined as disturbance, especially a random and persistent kind that obscures or reduces the clarity of a signal. In natural phenomena, noise could be induced by the process itself, such as local and intermittent instabilities, irresolvable sub-grid phenomena, or some concurrent processes in the environment in which the investigations are conducted. It could also be generated by the sensors and recording systems when observations are made. When efforts are made to understand data, important differences must be considered between the clean signals that are the direct results of the underlying fundamental physical processes of our interest ("the truth") and the noise induced by various other processes that somehow must be removed. In general, all data are amalgamations of signal and noise, i. e.,

$$\boldsymbol{x(t) = s(t) + n(t)}, \tag{1}$$

in which $\boldsymbol{x(t)}$ is the recorded data, and $\boldsymbol{s(t)}$ and $\boldsymbol{n(t)}$ are the true signal and noise, respectively. Because noise is ubiquitous and represents a highly undesirable and dreaded part of any data, many data analysis methods were designed specifically to remove the noise and extract the true signals in data, although often not successful.

Since separating the signal and the noise in data is necessary; three important issues should be addressed: 1) the dependence of the results on the analysis methods used and assumptions made on the data. (For example, a linear regression of data implicitly assumes the underlying physics of the data to be linear, while a spectrum analysis of data implies the process is stationary.) 2) The noise level to be tolerated in the extracted “signals”, for no analysis method is perfect, and in almost all cases the extracted “signals” still contain some noise. And, 3) the portion of real signal obliterated or deformed through the analysis processing as part of the noise. (For example, Fourier filtering can remove harmonics through low-pass filtering and thus deform the waveform of the fundamentals.)

All these problems cause misinterpretation of data, and the latter two issues are specifically related to the existence and removal of noise. As noise is ubiquitous, steps must be taken to insure that any meaningful result from the analysis should not be contaminated by noise. To avoid possible illusion, the null hypothesis test against noise is often used with the known noise characteristics associated with the analysis method (Wu and Huang 2004, 2005, Flandrin et al. 2005). Although most data analysis techniques are designed specifically to remove noise, there are, however, cases when noise is added in order to help data analysis, to assist the detection of weak signals, and to delineate the underlying processes. The intention here is to provide a brief survey of the beneficial utilization of noise in data analysis.

The earliest known utilization of noise in aiding data analysis was due to Press and Tukey (1956) known as pre-whitening, where white noise was added to flatten the narrow spectral peaks in order to get a better spectral estimation. Since then, pre-whitening has become a very common technique in data analysis. For example, Fuenzalida and Rosenbluth (1990) added noise to process climate data; Link and Buckley (1993) and Zala et al. (1995) used noise to improve acoustic signal. Strickland and Il Hahn (1997) used wavelet and added noise to detect objects in general, and Trucco (2001) used noise to help design special filters for detecting embedded objects on the ocean floor experimentally. Some general problems associated with this approach can be found in Priestley (1992), Kao et al. (1992), Politis (1993), and Douglas et al. (1999).

Another category of popular use of noise in data analysis is more related to the analysis method than to help extracting the signal from the data. Adding noise to data helps to understand the sensitivity of an analysis method to noise and the robustness of the results obtained. This approach is used widely, for example, Cichocki and Amari (2002) added noise to various data to test the robustness of the independent component analysis (ICA) algorithm, and De Lathauwer et al. (2005) used noise to identify error in ICA.

Adding noise to the input to specifically designed nonlinear detectors could be also beneficial to detecting weak periodic or quasi-periodic signals based on a physical process called stochastic resonance. The study of stochastic resonance was pioneered by Benzi and his colleagues in early 1980's. The details of the development of the theory of stochastic resonance and its applications can be found in a lengthy review paper by Gammaitoni et al. (1998). It should be noted here that *most of the past applications (including these mentioned earlier) have not used the cancellation effects associated with an ensemble of noise-added cases to improve their results.*

Specific to analysis using the Empirical Mode Decomposition, Huang et al. (2001) added infinitesimal magnitude noise to earthquake data in an attempt to prevent the low frequency mode from expanding into the quiescent region. But they failed to realize fully the implications of the added noise in the EMD method. The true advances related to the EMD method had to wait until the two pioneering works by Gledhill (2003) and Flandrin et al. (2005).

Flandrin et al. (2005) used added noise to overcome one of the difficulties of the original EMD method. As the EMD is solely based on the existence of extrema (either in amplitude or in curvature), the method ceases to work if the data lacks the necessary extrema. An extreme example is in the decomposition of a Dirac pulse (delta function), where there is only one extrema in the whole data set. To overcome the difficulty, Flandrin et al. (2005) suggested adding noise with infinitesimal amplitude to the Dirac pulse so as to make the EMD algorithm operable. Since the decomposition results are sensitive to the added noise, Flandrin et al. (2005) ran an ensemble of 5000 decompositions, with different versions of noise, all of infinitesimal amplitude. Though he used the mean as the final decomposition of the Dirac pulse, he defined the true answer as

$$d[n] = \lim_{e \to 0^+} E\{ d[n] + \varepsilon\, r_k[n] \} , \tag{2}$$

in which, $[n]$ represents nth data point, $d[n]$ is the Dirac function, $r_k[n]$ is a random number, ε is the infinitesimal parameter, and $E\{\ \}$ is the expected value. Flandrin's novel use of the added noise has made the EMD algorithm operable for a data set that could not be previously analyzed.

Another novel use of noise in data analysis is by Gledhill (2003), who used noise to test the robustness of the EMD algorithm. Although an ensemble of noise was used, he never used the cancellation principle to define the ensemble mean as the true answer. Based on his discovery (that noise could cause the EMD to produce slightly different outcomes), he assumed that the result from the clean data without noise was the true answer and thus designated it as the reference. He then defined the discrepancy, Δ, as

$$\Delta = \sum_{j=1}^{m} \left(\sum_t \left(cr_j(t) - cn_j(t) \right)^2 \right)^{1/2} , \tag{3}$$

in which cr_j and cn_j are the j^{th} component of the IMF without and with noise added, and m is the total number of IMFs generated from the data. In his extensive study of the detailed distribution of the noise-caused 'discrepancy', he concluded that the EMD algorithm is reasonably stable for small perturbations. This conclusion is in slight conflict with his observations that the perturbed answer with infinitesimal noise showed a bi-modal distribution of the discrepancy.

Gledhill had also pushed the noise-added analysis in another direction: He had proposed to use an ensemble mean of noise-added analysis to form a 'Composite Hilbert spectrum'. As the spectrum is non-negative, the added noise could not cancel out. He then proposed to keep a noise only spectrum and subtract it from the full noise-added spectrum at the end. This non-cancellation of noise in the spectrum, however, forced Gledhill (2003) to limit the noise used to be of small magnitude, so that he could be sure that there would not be too much interaction between the noise-added and the original clean signal, and that the contribution of the noise to the final energy density in the spectrum would be negligible.

Although the infinitesimal noise used by Gledhill (2003) had improved the confidence limit of the final spectrum, this work neither fully explored the cancellation property of the noise, nor the power of finite perturbation to explore all possible solutions. Furthermore, it is well known that whenever there is intermittence, the signal without noise can produce IMFs with mode mixing. There is no justification to assume the result without added noise is the truth or the reference signal. These reservations notwithstanding, all these studies by Flandrin et al. (2005) and Gledhill (2003) had still greatly advanced the understanding of the effects of noise in the EMD method, though the crucial effects of noise had yet to be clearly articulated and fully explored.

In the following, the new noise-added EMD approach will be explained, in which the cancellation principle will be fully utilized, even with finite amplitude noise. Also emphasized is the finding that the true solution of the EMD method should be the ensemble mean rather than the clean data. This full presentation of the new method will be the subject of the next section.

3. Ensemble Empirical Mode Decomposition

3.1 The Empirical Mode Decomposition

This section starts with a brief review of the original EMD method. The detailed method can be found in Huang et al. (1998) and Huang et al. (1999). Different to almost all previous methods of data analysis, the EMD method is adaptive, with the basis of the decomposition based on and derived from the data. In the EMD approach, the data $\boldsymbol{X(t)}$ is decomposed in terms of IMFs, $\boldsymbol{c_j}$, i.e.,

$$x(t) = \sum_{j=1}^{n} c_j + r_n \ , \tag{4}$$

where $\boldsymbol{r_n}$ is the residue of data $\boldsymbol{x(t)}$, after $\boldsymbol{n}$ number of IMFs are extracted. IMFs are simple oscillatory functions with varying amplitude and frequency, and hence have the following properties:

1. throughout the whole length of a single IMF, the number of extrema and the number of zero-crossings must either be equal or differ at most by one (although these numbers could be differ significantly for the original data set);
2. at any data location, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

In practice, the EMD is implemented through a sifting process that uses only local extrema. From any data $\boldsymbol{r_{j-1}}$, say, the procedure is as follows: 1) identify all the local extrema (the combination of both maxima and minima) and connect all these local maxima (minima) with a cubic spline as the upper (lower) envelope; 2) obtain the first component $\boldsymbol{h}$ by taking the difference between the data and the local mean of the two envelopes; and 3) treat $\boldsymbol{h}$ as the data and repeat steps 1 and 2 as many times as is required until the envelopes are symmetric with respect to zero mean under certain criteria. The final $\boldsymbol{h}$ is designated as $\boldsymbol{c_j}$. A complete sifting process stops when the residue, $\boldsymbol{r_n}$, becomes a monotonic function from which no more IMF can be extracted.

Based on this simple description of EMD, Flandrin et al (2004) and Wu and Huang (2004) have shown that, if the data consisted of white noise which has scales populated uniformly through the whole time-scale or time-frequency space, the EMD behaves as a dyadic filter bank: the Fourier spectra of various IMFs collapse to a single shape along the axis of logarithm of period or frequency. Then the total number of IMFs of a data set is close to $\boldsymbol{log_2 N}$ with $\boldsymbol{N}$ the number of total data points. When the data is not pure noise; some scales could be missing, and that is when the mode mixing phenomenon occurs.

3.2 Mode Mixing Problem

'Mode mixing" is defined as any IMF consisting of oscillations of dramatically disparate scales, mostly caused by intermittency of the driving mechanisms. When mode mixing occurs, an IMF can cease to have physical meaning by itself, suggesting falsely that there may be different physical processes represented in a mode. Even though the final time-frequency projection could rectify the mixed mode to some degree, the alias at each transition from one scale to another would irrecoverably damage the clean separation of scales. Such a drawback was first illustrated in Huang et al. (1999) in which the modeled data was a mixture of intermittent high frequency oscillations riding on a continuous low frequency sinusoidal signal. An almost identical example used by Huang et al. (1999) is presented here in detail as an illustration.

The data and its sifting process are illustrated in Figure 1. The data has its fundamental part as a low frequency sinusoidal wave with unit amplitude. At the three middle crests of the low frequency wave, high frequency intermittent oscillations with amplitude of 0.1 are riding on the fundamental, as panel ***a*** of Figure 1 shows. The sifting process starts with identifying the maxima (minima) in the data. In this case, 15 local maxima are identified, with the first and the last coming from the fundamental, and the other 13 caused mainly by intermittent oscillations (panel ***b***). As a result, the upper envelope resembles neither the upper envelope of the fundamental (which is a flat line at one) nor the upper one of the intermittent oscillations (which is supposed to be the fundamental outside intermittent areas). Rather, the envelope is a severely distorted combination of both (the red line in panel ***c***). Consequently, the initial guess of the first IMF (panel ***d***) is the mixture of both the low frequency fundamental and the high frequency intermittent waves, as shown in Figure 2, making it more difficult to interpret and identify the underlying physical processes.

To alleviate this drawback, Huang et al. (1999) proposed an intermittence test that subjectively extracts the oscillations with periods significantly smaller than a pre-selected value during the sifting process. The method works quite well for this example. However, for complicated data with scales variable but continuously distributed, no single criterion of intermittence test can be selected. Furthermore, the most troublesome aspect of this subjectively pre-selected criterion is that it lacks physical justifications and renders the EMD non-adaptive. Additionally, mode mixing is also the main reason that renders the EMD algorithm unstable: Any small perturbation may result in a new set of IMFs as reported by Gledhill (2003). Obviously, the intermittence prevents EMD from extracting any signal with similar scales. To solve these problems, the EEMD is proposed, which will be described in the following.

3.3 *Ensemble Empirical Mode Decomposition*

As given in Equation (1), all data are amalgamations of signal and noise. To improve the accuracy of measurements, the ensemble mean is a powerful approach, where data are collected by separate observations, each of which contains different noise. To generalize this ensemble idea, noise is introduced to the single data set, $\boldsymbol{x(t)}$, as if separate observations were indeed being made as an analogue to a physical experiment that could be repeated many times. The added white noise is treated as the possible random noise that would be encountered in the measurement process. Under such conditions, the $\boldsymbol{i^{th}}$ 'artificial' observation will be

$$\boldsymbol{x_i(t) = x(t) + w_i(t)} \,. \tag{5}$$

In the case of only one observation, one of the multiple-observation ensembles is mimicked by adding not arbitrary but different copies of white noise, $\boldsymbol{w_i(t)}$, to that single observation as given in Equation (5). Although adding noise may result in smaller signal-to-noise ratio, the added white noise will provide a uniform reference scale distribution to facilitate EMD; therefore, the low signal-noise ratio does not affect the decomposition method but actually enhances it to avoid the mode mixing. Based on this argument, an additional step is taken by arguing that adding white noise may help to extract the true signals in the data, a method that is termed Ensemble Empirical Mode Decomposition (EEMD), a truly *noise-assisted data analysis* (NADA) method.

Before looking at the details of the new EEMD, a review of a few properties of the original EMD is presented:

1. the EMD is an adaptive data analysis method that is based on local characteristics of the data, and hence, it catches nonlinear, non-stationary oscillations more effectively;

2. the EMD is a dyadic filter bank for any white (or fractional Gaussian) noise-only series.
3. when the data is intermittent, the dyadic property is often compromised in the original EMD as the example in Figure. 2 shows;
4. adding noise to the data could provide a uniformly distributed reference scale, which enables EMD to repair the compromised dyadic property; and
5. corresponding IMFs of different series of noise have no correlation with each other. Therefore, the means of the corresponding IMFs of different white noise series are likely to cancel each other.

With these properties of the EMD in mind, the proposed Ensemble Empirical Mode Decomposition is developed as follows:

1. add a white noise series to the targeted data;
2. decompose the data with added white noise into IMFs;
3. repeat step 1 and step 2 again and again, but with different white noise series each time; and
4. obtain the (ensemble) means of corresponding IMFs of the decompositions as the final result.

The effects of the decomposition using the EEMD are that the added white noise series cancel each other, and the mean IMFs stays within the natural dyadic filter windows, significantly reducing the chance of mode mixing and preserving the dyadic property.

To illustrate the procedure, the data in Figure 1 is used as an example. If the EEMD is implemented with the added noise having an amplitude of 0.1 standard deviation of the original data for just one trial, the result is given in Figure 3. Here the low frequency component is already extracted almost perfectly. The high frequency components, however, are buried in noise. Note that when the number of ensemble members increases, high frequency intermittent signal emerges, as Figure 4 displays. Clearly, the fundamental signal (C5) is represented nearly perfect, as well as the intermittent signals, if C2 and C3 are added together. This provides the first example to demonstrate that the noise-assisted data analysis using the EEMD significantly improves the capability of extracting signals in the data, and represents a major improvement of the EMD method.

It should be pointed out that the effect of the added white noise should decrease following the well-established statistical rule:

$$\boldsymbol{\varepsilon_n} = \frac{\boldsymbol{\varepsilon}}{\sqrt{\boldsymbol{N}}}, \tag{6a}$$

or

$$\boldsymbol{ln}\,\varepsilon_n + \frac{\boldsymbol{\varepsilon}}{2}\boldsymbol{ln}\,\boldsymbol{N} = 0, \tag{6b}$$

where $\boldsymbol{N}$ is the number of ensemble members, $\boldsymbol{\varepsilon}$ is the amplitude of the added noise, and $\boldsymbol{\varepsilon_n}$ is the final standard deviation of error, which is defined as the difference between the input signal and the corresponding IMF(s). Such a relation is clear in Figure 5, in which the standard deviation of error is plotted as a function of the number of ensemble members. In general, the results agree well with the theoretical prediction. The relatively large deviation for the fundamental signal from the theoretical line fitting is understandable: the spread of error for low frequency signals is large, as pointed by Wu and Huang (2004).

In fact, if the added noise amplitude is too small, then it may not introduce the change of extrema that the EMD relies on. However, by increasing the ensemble members, the effect of the added white noise will always be able to be reduced to a negligibly small level. Having demonstrated the basic approach, additional examples will be given of adding noise with finite (or even larger) amplitude in the next section, dealing with complicated real data.

4. More Examples

The previous example introduced the concept of noise-assisted data analysis using the EEMD method. The question now is whether the EEMD indeed helps in reaching the ultimate goal of data analysis: to isolate and extract the true signals in data, and thereby to understand the properties of data and its underlying physics. The easiest way to demonstrate the power of the EEMD and its usefulness is to apply to data of natural phenomena. In this section, EEMD is applied to two real cases: the first one is climate data that define the interaction between atmosphere and ocean; and the second one is a section of a high resolution digitalized sound record. Both cases are complicated and have rich properties in the data. These data are considered general enough to be the representatives of real cases.

4.1 Example 1: Analysis of Climate Data

The first set of data to be examined here is representative of an interacting air-sea system in the tropics known as the El Niño-Southern Oscillation (ENSO) phenomenon. The Southern Oscillation (SO) is a global-scale seesaw in atmospheric pressure between the western and the southeastern tropical Pacific, and the El Niño refers to variations in temperature and circulation in the tropical Pacific Ocean. The two systems are closely coupled (Philander, 1990, National Research Council 1996), and together they produce important climate fluctuations that have a significant impact on weather and climate over the globe as well as social and economic consequences (see, for example, Glantz et al. 1991). The underlying physics of ENSO have been explained in numerous papers (see, e.g., Suarez and Schopf 1988, Battisti and Hirst 1989, Jin 1996).

The Southern Oscillation is often represented by the Southern Oscillation Index (SOI), a normalized monthly sea level pressure index based on the pressure records collected in Darwin, Australia and Tahiti Island in the eastern tropical Pacific (Trenberth 1984). It should be noted here that the Tahiti record used for the calculation of the SOI is less reliable and contains missing data prior to 1935. The Cold Tongue Index (CTI), defined as the average large year-to-year SST anomaly fluctuations over 6°N-6°S, 180-90°W, is a good representation of El Niño (Deser and Wallace 1990). A large negative peak of SOI, which often occurs with a two to seven year period, corresponds to a strong El Niño (warm) event. With its rich statistical properties and scientific importance, the SOI is one of the most prominent time series in the geophysical research community and has been well studied. Many time series analysis tools have been used on this time series to display their capability of revealing useful scientific information (e.g., Wu et al. 2001, Ghil et al. 2002, Wu and Huang 2004). The specific question to be examined here is over what time scales are the El Niño and the Southern Oscillation coupled?

The SOI used in this study is described in Ropelewski and Jones (1987) and Allan et al. (1991). The CTI is based on the SST from January 1870 to December 2002 provided by the Hadley Center for Climate Prediction and Research (Rayner et al., 1996), which is refined from direct observations. The sparse and low quality observations in the early stages of the period makes the two indices in the early stages less consistent and their inter-relationship less reliable, as reflected by the fact that the overall correlations between the two time series is -0.57 for the whole data length, but only -0.45 for the first half, and -0.68 for the second half. The two indices are plotted in Figure. 6.

The decompositions of these two indices using the original EMD are plotted in Figure 7. Although SOI and CTI have a quite large correlation (-0.57), their corresponding IMFs, however, show little synchronization. For the whole data length, the largest negative correlation amongst the IMFs is only -0.43 (see Figure 8), a much smaller value than that of the correlation between the whole data of SOI and CTI. Since the underlying physical processes that dictate the large scale interaction between atmosphere and ocean differ on various timescales, a good decomposition method is expected to identify such variations. However, the low correlations between corresponding IMFs seems to indicate that the decompositions using the original EMD on SOI and CTI helps little to identify and understand which times cales for the coupling between atmosphere and ocean in climate system in the tropics are more prominent.

This lack of correlation clearly represents a typical problem of mode mixing in the original EMD. From a visual inspection, it is easily seen that in almost any high or middle scale IMF of SOI or CTI, pieces of oscillations having approximate periods of those appear also in its neighboring IMFs. The mixing is also contagious: if it happens in one IMF, it will happen in the following IMFs at the same temporal neighborhood. Consequently, mode mixing reduces the capability of the EMD in identifying the true time scales of consistent coupled oscillations in the individual IMF component in the

ENSO system. This is clearly shown in Figure 8, in which no IMF pairs with a rank from 1 to 7 have a higher correlation than the full data set.

To solve this problem and to identify the time scale at which the interaction truly occurs, both time series were reanalysed using the EEMD. The results are displayed in Figure 9. It is clear that the synchronizations between corresponding IMFs pairs are much improved, especially for the IMF components 4-7 in the later half of the record. As mentioned earlier, both SOI and CTI are not as reliable in the first half of the record as those in the second half due to the sparse or missing observations. Therefore, the lower degree of synchronization of the corresponding IMF components of SOI and of CTI in the earlier half is not likely caused by EEMD, but by the less consistent data of SOI and CTI in that period. To quantify this claim, the detailed correlation values of corresponding IMF pairs will be discussed next.

The detailed correlation between corresponding IMF components of SOI and CTI are displayed in Figure 10. Clearly, the decompositions using the EEMD improve the correlation values significantly. The EEMD results help greatly in the isolation of signals of various scales that reflect coupling between atmosphere and ocean in the ENSO system. Consistently high correlations between IMFs from SOI and CTI on various timescales have been obtained, especially those of interannual (Components 4 and 5 with mean periods of 2.83 and 5.23 years respectively) and short interdecadal (components 6 and 7 with mean periods of 10.50 and 20.0 years respectively) timescales. The increase of the correlation coefficients from just under 0.68 for the later half of the whole data to significantly over 0.8 for these IMF pairs is remarkable. There has not yet been any other filtering method used to study these two time series that has led to such high correlations between the band-filtered results from published literature on all these timescales. (For the long interdecadal time scales, especially for C8 and C9, since the number of degree of freedom of the IMF components is very small due to the lack of oscillation variations, the correlation coefficients corresponding to them can be very misleading; therefore, they should be ignored.) These results clearly indicate the most important coupling between the atmosphere and the ocean occurs on a broad range of timescales, covering interannual and interdecadal scales from 2 to 20 years.

The high correlations on interannual and short interdecadal timescales between IMFs of SOI and CTI, especially in the latter half of the record, are consistent with the physical explanations provided by recent studies. These IMFs are statistically significant at 95% confidence level based on a testing method proposed in Wu and Huang (2004, 2005) against the white noise null hypothesis. The two inter-annual modes (C4 and C5) are also statistically significant at 95% confidence level against the traditional red noise null hypothesis. Indeed, Jin et al. (personal communications, their manuscript being under preparation) has solved a nonlinear coupled atmosphere-ocean system and showed analytically that the interannual variability of ENSO has two separate modes with periods in agreement with the results obtained here. Concerning the coupled short interdecadal modes, they are also in good agreement with a recent modeling study by Yeh and Kirtman (2004), which demonstrated that such modes can be a result of a coupled system in response to stochastic forcing. Therefore, the EEMD method does provide a more accurate tool to isolate signals with specific time scales in observational data produced by different underlying physics.

The sensitivity of the decomposition of data using the EEMD to the amplitude of noise will be examined. In Figs. 11 and 12, noise with a standard deviation of 0.1, 0.2, and 0.4 is added. The ensemble size for each case is 100. Clearly, the synchronization between cases of different levels of added noise is remarkably good, except the case of no noise added, when mode mixing produced an unstable decomposition. In the latter case, any perturbation may push the result to a different state as studied by Gledhill (2003). Additionally, the improvement of the decomposition for the CTI seems to be greater than that for the SOI. The reason is simple: SOI is much noisier than CTI, since the former is based on noisy observations of sea level data from only two locations (Darwin and Tahiti pressures), while CTI is based on the averaged observed sea surface temperature at hundreds of locations along the equator. This indeed indicates that EMD is a noise-friendly method: the noise contained in the data makes the EMD decomposition truly dyadic.

More decomposition of SOI and CTI with various noise levels and ensemble members has been carried out. The results (not shown here) indicate that increasing noise amplitudes and ensemble numbers alter the decomposition little as long as added noise has moderate amplitude and the ensemble has a large enough number of trials. It should be noticed that the number of ensemble numbers should increase when the amplitude of noise increases so as to reduce the contribution of added noise in the decomposed results. The conclusions drawn for decompositions of SOI and CTI here are also true for other data tried with the EEMD method. Therefore, the EEMD provides a sort of “uniqueness” and robustness result that the original EMD usually could not, and it also increases the confidence of the decomposition.

4.2 Example 2: Analysis of Voice Data

In the previous example, the demonstration of power and confirmation of the usefulness of the EEMD were made through analyzing two different but physically closely interacted subsystems (corresponding to two different data sets) of a climate system. Such a pair of highly related data sets is rare in more general cases of signal processing. Therefore, to further illustrate the EEMD as an effective data analysis method in time-frequency domain for general purpose, we analyze a piece of speech data using the EEMD. The original data, given in Figure 13, shows the digitalized sound of the word, 'Hello', at 22,050 Hz digitization (Huang 2003).

The EMD components obtained from the original EMD without added noise is given in Figure 14 plotted with a uniform scale. Here we can see very clear mode mixing from the second component and down, where high disparate amplitudes and scales are obvious. The mode mixing influences the scale parity in all the IMF components, though some are not as obvious.

The same data was then processed with the EEMD with a noise selected at an amplitude of 0.2 times that of the data RMS, and 1000 trials. The result is a drastic improvement as shown in Figure 15. Here all the IMF components are continuous and without obvious fragmentation. The third IMF is almost the full signal, which can produce a sound that is clear and with almost the original audio quality. All other components are also regular and have comparable and uniform scales and amplitudes for each respective IMF component, but the sounds produced by them are not intelligible: they mostly consist of either high frequency hissing or low frequency moaning. The results once again clearly demonstrate that the EEMD has the capability of catching the essence of data that manifests the underlying physics.

The improvements on the quality of the IMFs also have drastic effects on the time-frequency distribution of the data in Hilbert spectra format as shown in the two different Hilbert spectra given in Figures 16 and 17. In the original EMD, the mode mixings have caused the time-frequency distribution to be fragmented in Figure 16. The alias at the transition points from one scale to another is clearly visible. Although the Hilbert spectra of this quality could be used for some general purposes such as identifying the basic frequencies and their ranges of variation, quantitative measures would be extremely difficult. The Hilbert spectrum from the EEMD given in Figure 17, however, shows a great improvement. All the basic frequencies are continuous with no transitional gaps.

5. Discussion and Conclusions

The basic principle of the Ensemble Empirical Mode Decomposition (EEMD) is simple, yet the power of this new approach is obvious from the examples. The new method indeed can separate signals of different scales without undue mode mixing. Adding white noise helps to establish a dyadic reference frame in the time-frequency or time-scale space. The real data with a comparable scale can find a natural location to reside. The EEMD utilizes all the statistical characteristic of the noise: It helps to perturb the signal and enable the EMD algorithm to visit all possible solutions in the finite (not infinitesimal) neighborhood of the true final answer; it also takes advantage of the zero mean of the noise to cancel out this noise background once it has served its function of providing the uniformly distributed frame of scales, a feat only possible in the time domain data analysis. In a way, this new approach is essentially a controlled repeated experiment to produce an ensemble mean for a non-stationary data as the final answer. Since the role of the added noise in the EEMD is to facilitate the separation of different scales of the inputted data without a real contribution to the IMFs of the data, the EEMD is a truly noise-assisted data analysis (NADA) method that is effective in extracting signals from the data.

Although the noise-added analysis has been tried by the pioneers such as Flandrin et al. (2004) and Gledhill (2003), there are crucial differences between our approach and theirs. First, both Flandrin and Gledhill define the truth either as the results without noise added, or as given in Equation (2), which is the limit when the noise-introduced perturbation approaches zero. The truth defined by EEMD is given by the number in the ensemble approaching infinity, i.e.

$$c_j(t) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \left\{ c_j(t) + \alpha\, r_k(t) \right\}, \tag{7}$$

in which

$$c_j(t) + \alpha\, r_k(t), \tag{8}$$

is the k^{th} trial of the j^{th} IMF in the noise-added signal, and the magnitude of the added noise, α, is not necessarily small. But the number of the trials in the ensemble, N, has to be large. The difference between the truth and the result of the ensemble is governed by the well-known statistical rule: it decreases as one over the square-root of N, as given in Equation (6).

With the truth defined, the discrepancy, Δ, instead of the one given in Equation (3), should be

$$\Delta = \sum_{j=1}^{m} \left(\sum_{t} \left(E\left\{ cn_j(t) \right\} - cn_j(t) \right)^2 \right)^{1/2} , \tag{9}$$

in which ***E{ }*** is the expected value as given in Equation (7).

It is proposed here that the EEMD indeed represents a major improvement over the original EMD. As the level of added noise is not of critical importance, as long as it is of finite amplitude to enable a fair ensemble of all the possibilities, the EEMD can be used without any subjective intervention; thus, it provides a truly adaptive data analysis method. By eliminating the problem of mode mixing, it also produces a set of IMFs that bears the full physical meaning, and a time-frequency distribution without transitional gaps. The EMD, with the ensemble approach, has become a more mature tool for nonlinear and non-stationary time series (and other one dimensional data) analysis.

While the EEMD offers great improvement over the original EMD, there are still some unsettled problems. The first one is a drawback of the EEMD: the EEMD produced results do not satisfy the strict definition of IMF. Although each trial in the ensemble produces a set of IMF components, the sum of IMF is not necessarily an IMF. The deviations from strict IMFs, however, are small for the examples presented in this study, and have not interfered noticeably in the computation of instantaneous frequency using Hilbert Transform or any other methods as discussed by Huang et al. (2005). Nevertheless, these imperfections should be eliminated. One possible solution is to conduct another round of sifting on the IMFs produced by the EEMD. As the IMFs results from the EEMD are of comparable scales, mode mixing would not be a critical problem here; and a simple sift could separate the riding waves without any problem. This topic will be discussed and reported elsewhere.

The second problem associated with the EEMD is how to treat multi-mode distribution of the IMFs. As discussed by Gledhill (2003), the discrepancy between a trial and its reference tends to show a bi-modal (if not multi-modal) distribution. Whenever a bi-modal distribution occurs, the discrepancy values could be quite large and the variance value would no longer follow the formula given by Equation (6). Although part of the large discrepancy could be possibly attributed to the selection of reference as the un-perturbed state, the selection of the reference alone cannot explain all the variance and its distribution. The true cause of the problem may be explained easily based on the study of white noise using the EMD by Wu and Huang (2004), in which the dyadic filter bank shows some overlap in scales. Signals having a scale located in the overlapping region would have finite probability appearing in two different modes. Although the problem has not been fully resolved by far, some alternative implementations of the sifting procedures can alleviate its severity. The first alternative is to tune the noise level and use more trials to reduce root-mean-squared deviation. Gledhill's results clearly show that this is possible, for the 'bi-modal' distribution indeed tends to merge into a single, albeit wider, uni-modal distribution. The second alternative is the one used in majority cases in this study: sift a low but fixed number of times (10 in this study) for obtaining each IMF components. Constrained by the dyadic filter bank property of EMD, this method would almost guarantee the same number of IMFs being sifted out from each trial in the ensemble although the copies of added noise in various trials are different. Both approaches have been tried in this study, but none avoids the multi-mode problem totally. The true solution may have to combine the multi-mode into a single mode, and sift it again to produce a proper single IMF. The third approach is to use rigorous check of the each component against the definition, and divide the outcome into different groups according to the total number of IMF generated. Our limited experience is that the distribution of the number of IMF is quite narrow even with moderate amount of noise perturbation. Then the peak of the distribution is adopted as the answer. We found all the approaches acceptable, and their differences small. Further studies will be carried out on this issue.

Finally, our experience in using the EEMD brought up two other previously persisted problems for the EMD: the end effect and the stoppage criteria. Both of the problems have been long-existing and not solved yet. The confidence limit of the EMD produced results and its dependence on stoppage criteria have been addressed to some extent by Huang et al. (2003). Here the EEMD provides an alternative, but better, measure of confidence limit, since the EEMD produced decompositions are much less sensitive to the stoppage criteria used. However, the optimal range of stoppage criteria as given by Huang at al. (2003) can still be used as a guide here. As for the end effect, the noise added processes help to ameliorate the difficulty, for with the added noise the end slope will be more uniformly distributed. Thus the final results could avoid a deterministic drift in one direction or the other. Nevertheless, a more through study of the end effect is urgently needed.

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Kenneth Arnold of Cornell University for helping to clarify the consequence of dyadic filter behavior in multi-modal IMF results. Wu is grateful to Drs. E. K. Schneider, B. P. Kirtman, and J. L. Kinter of George Mason University and the Center for Ocean-Land-Atmosphere Studies for their helpful suggestions and encouragements. Wu was sponsored by NSF under grant ATM-0342104 and NEH, NASA basic research program.

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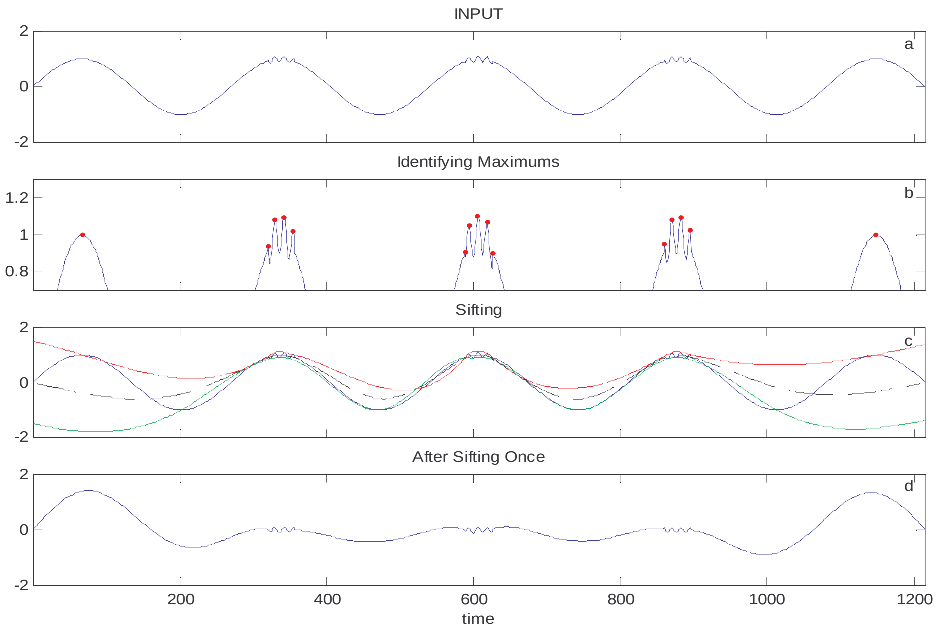

Figure 1: The very first sifting process. Panel ***a*** is the input; panel ***b*** identifies local maxima (red dots); panel ***c*** plots the upper envelope (red) and low envelope (blue) and their mean (black); and panel ***d*** is the difference between the input and the mean of the envelopes.

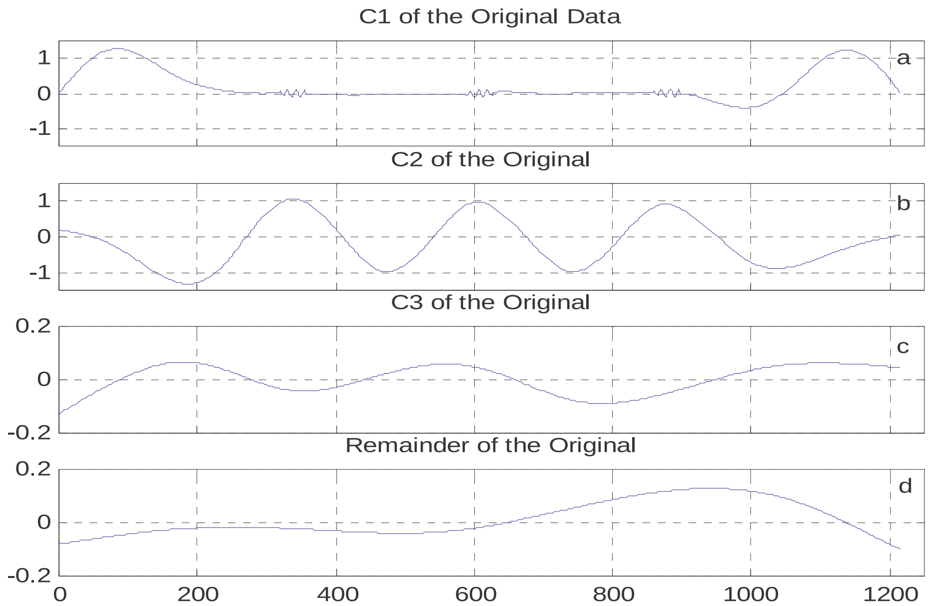

Figure 2: The intrinsic mode functions of the input displayed in Figure 1a.

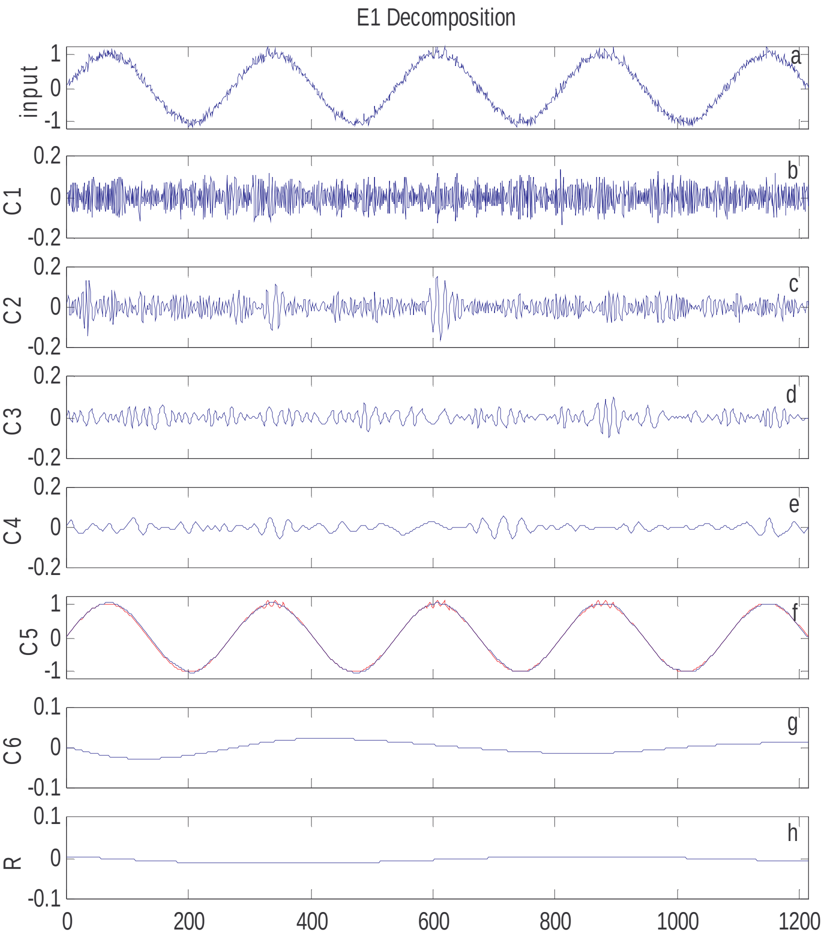

Figure 3: The intrinsic mode functions (panels b-g), and the trend of the modified input (panel a). In panel f, the original input is plotted in red as a comparison.

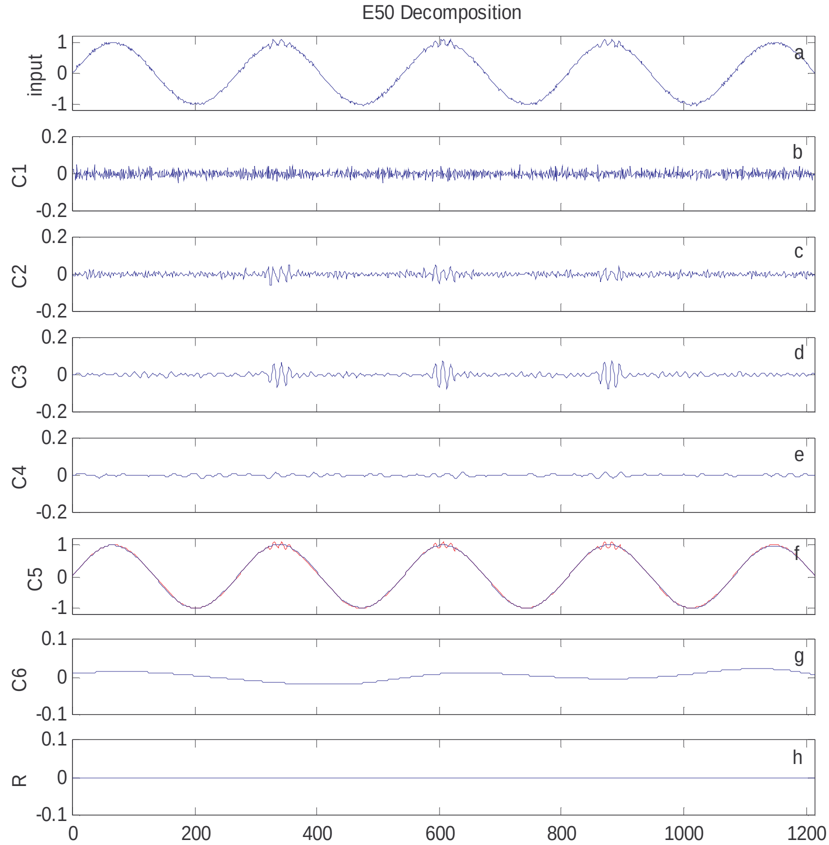

Figure 4: The IMF-like components of the decomposition of the original input in Figure 1a using the EEMD. In the EEMD, an ensemble member of 50 is used, and the added white noise in each ensemble member has a standard deviation of 0.1. In panel a, the mean of the noise modified input is plotted. In panel 5, the original input (red line) is also displayed for comparison.

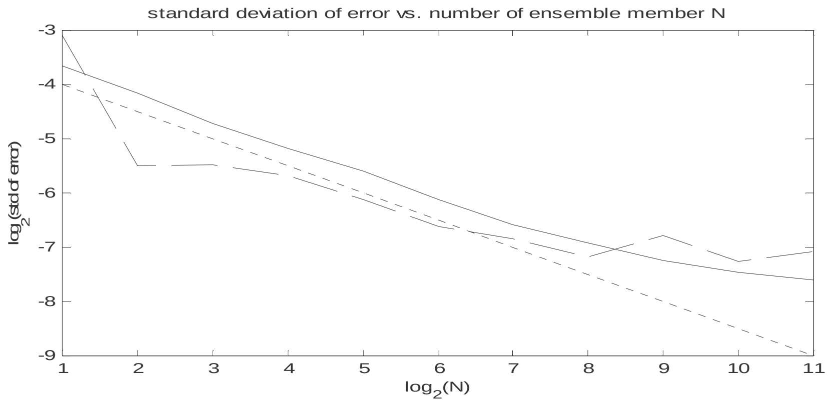

Figure 5: The standard deviation of error as a function of the number of ensemble members. The solid line is for the high frequency intermittent signals, and the dashed line is for the low frequency fundamental signals. The dotted line is the theoretical line predicted by Eq. (6) with arbitrary vertical location, used as a reference.

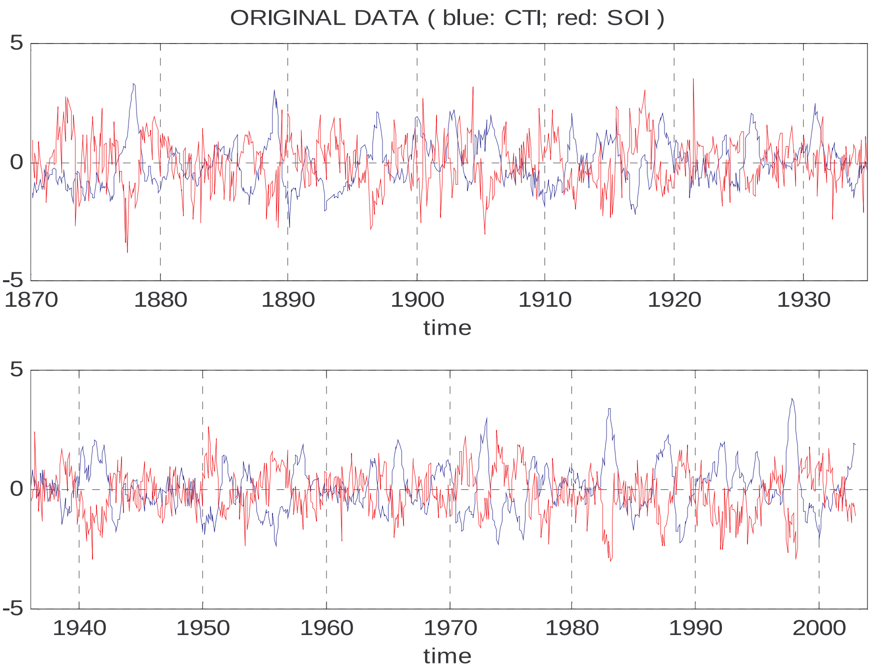

Figure 6: The Southern Oscillation Index (blue line) and the Cold Tongue Index (red line).

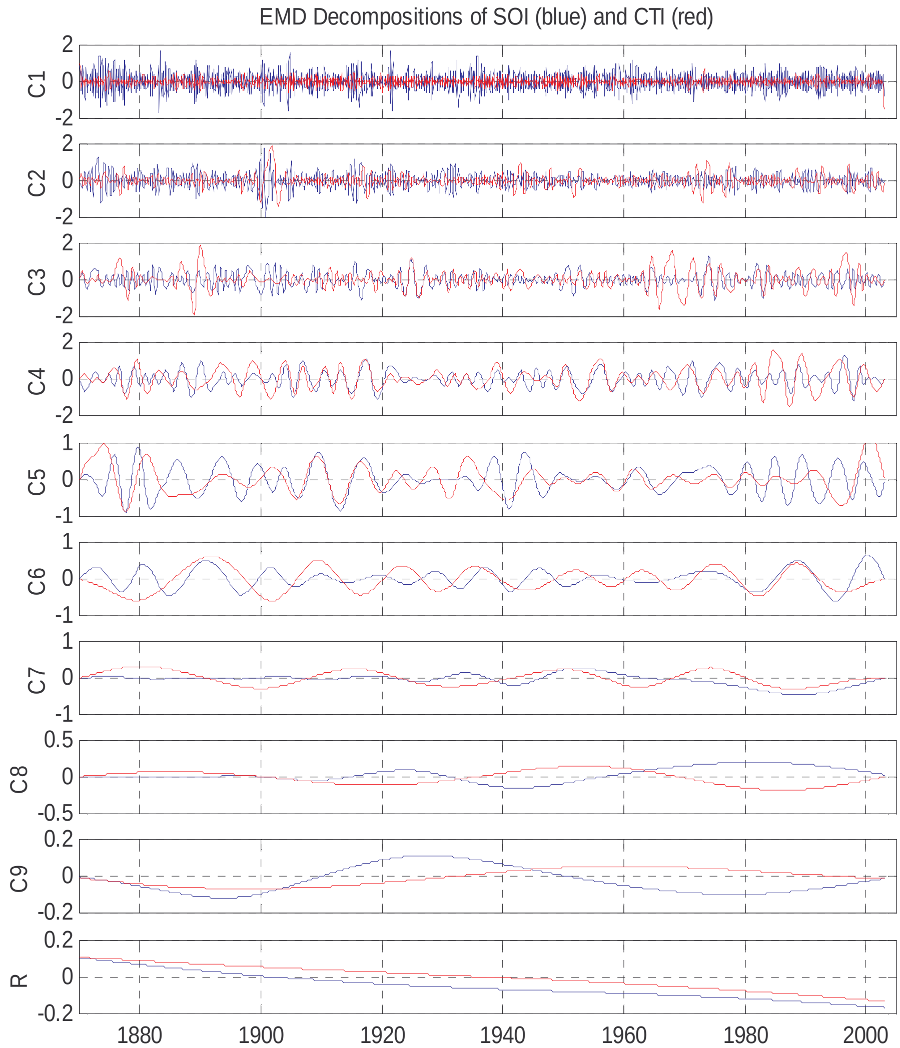

Figure 7: The intrinsic mode functions and the trends of the Southern Oscillation Index (blue lines) and the Cold Tongue Index (blue lines). For the convenience of identifying their synchronization, the CTI and its components are flipped.

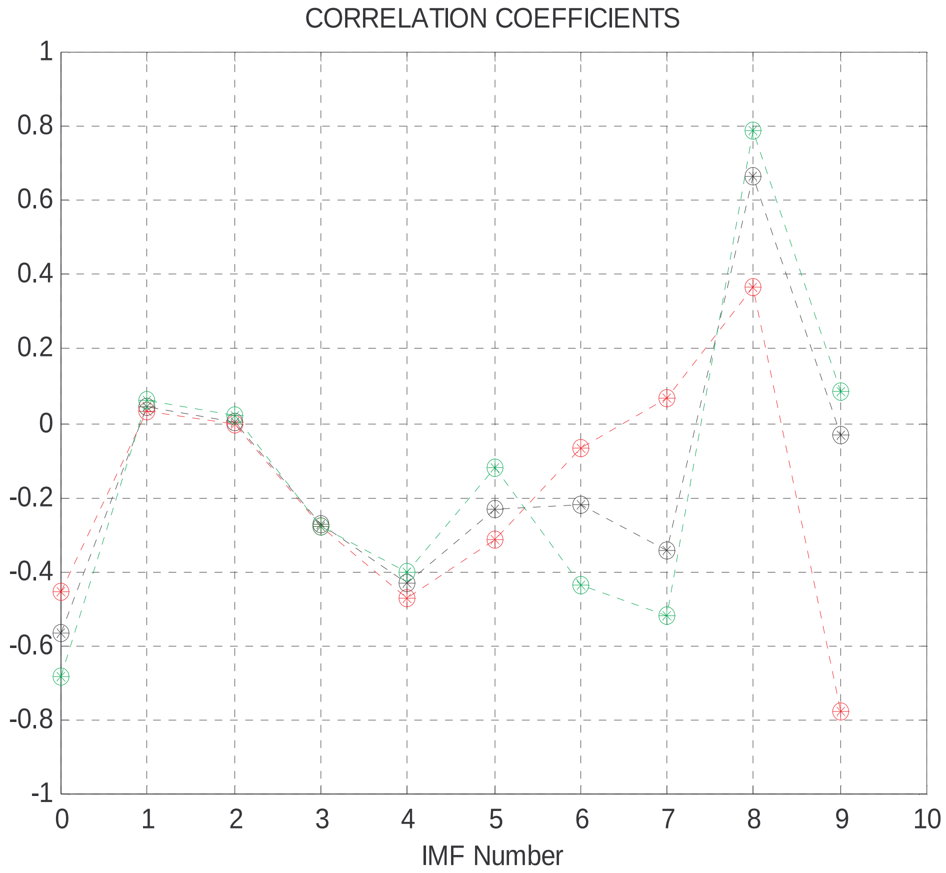

Figure 8: The correlation coefficients (asterisk-circle) of the SOI and CTI and their corresponding IMFs. IMF 0 here means the original signal. The black is for the whole data length; the red is for the first half; and the green for the second half.

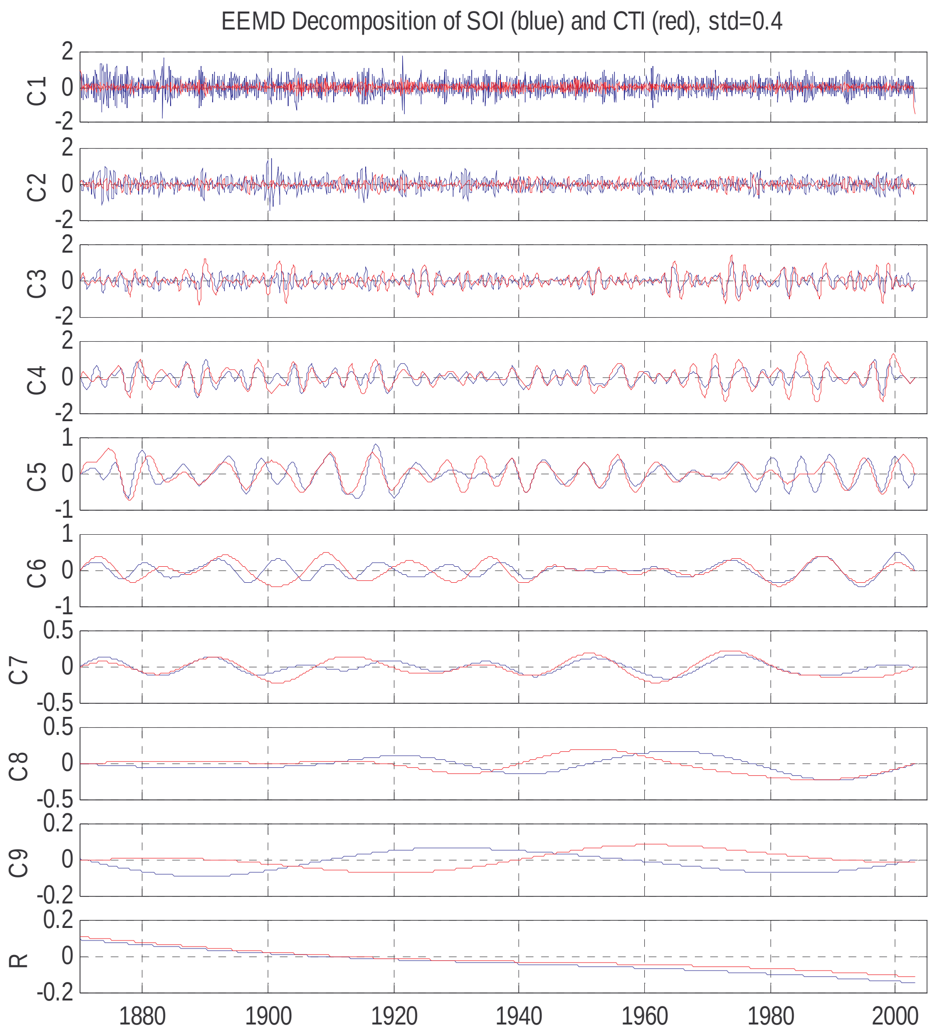

Figure 9: The IMF-like components of the decompositions of the SOI (blue lines) and (red lines) the CTI using the EEMD. In the EEMD, an ensemble size of 100 is used, and the added white noise in each ensemble member has a standard deviation of 0.4. For the convenience of identifying their synchronization, the CTI and its components are flipped.

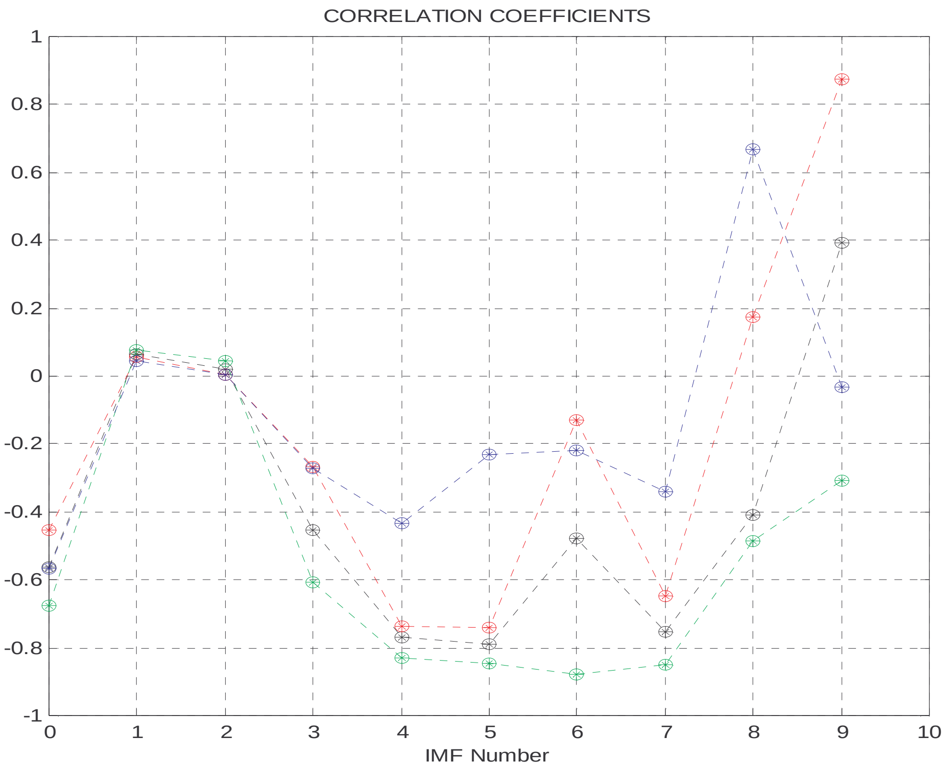

Figure 10: The correlation coefficients (asterisk-circle) of the SOI and CTI and their corresponding IMF-like components. IMF 0 here means the original signal. The black is for the whole data length; the red is for the first half; and the green for the second half. The blue is the same as the black in Figure 8, i.e., the correlation coefficients of the SOI and CTI and their corresponding IMFs for the whole data length.

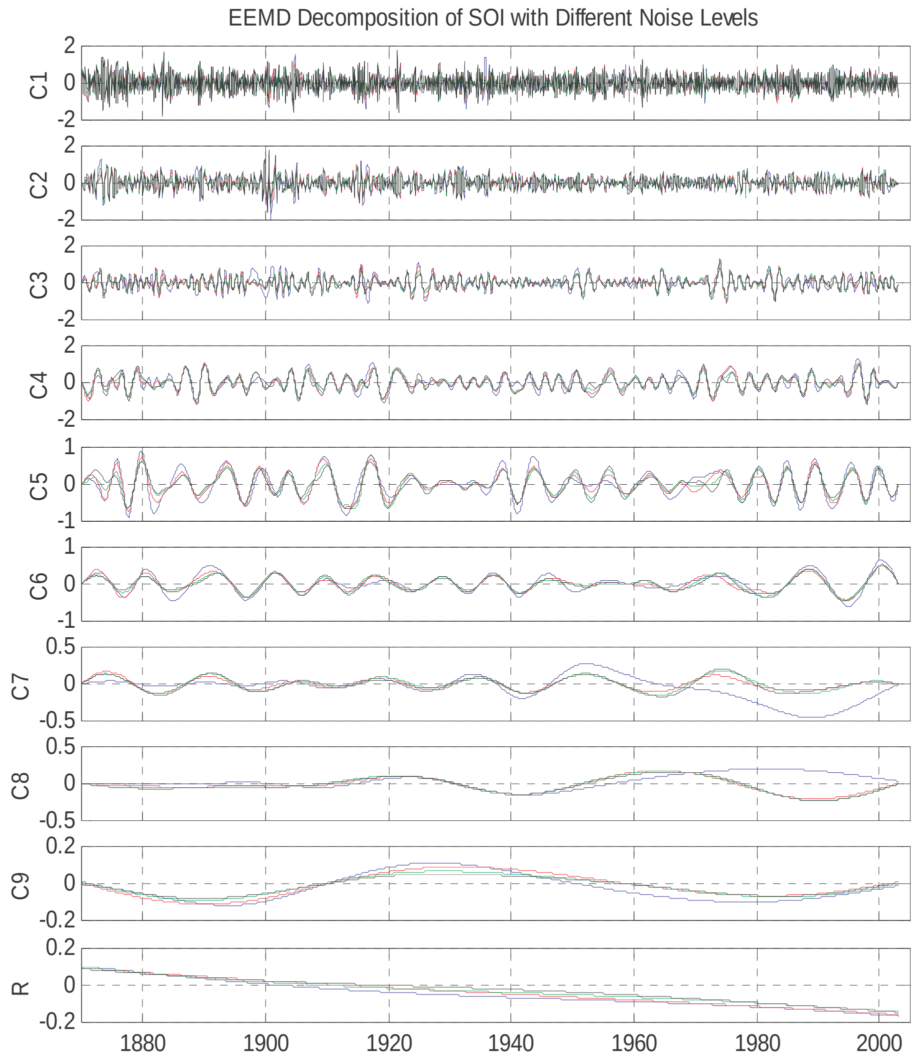

Figure 11: EEMD decompositions of SOI with added noise. Blue line corresponds is the standard decomposition using EMD without any added noise. Red, green, and black lines correspond to EEMD decompositions with added noise of standard deviation of 0.1, 0.2, and 0.4, respectively. The ensemble number for each case is 100.

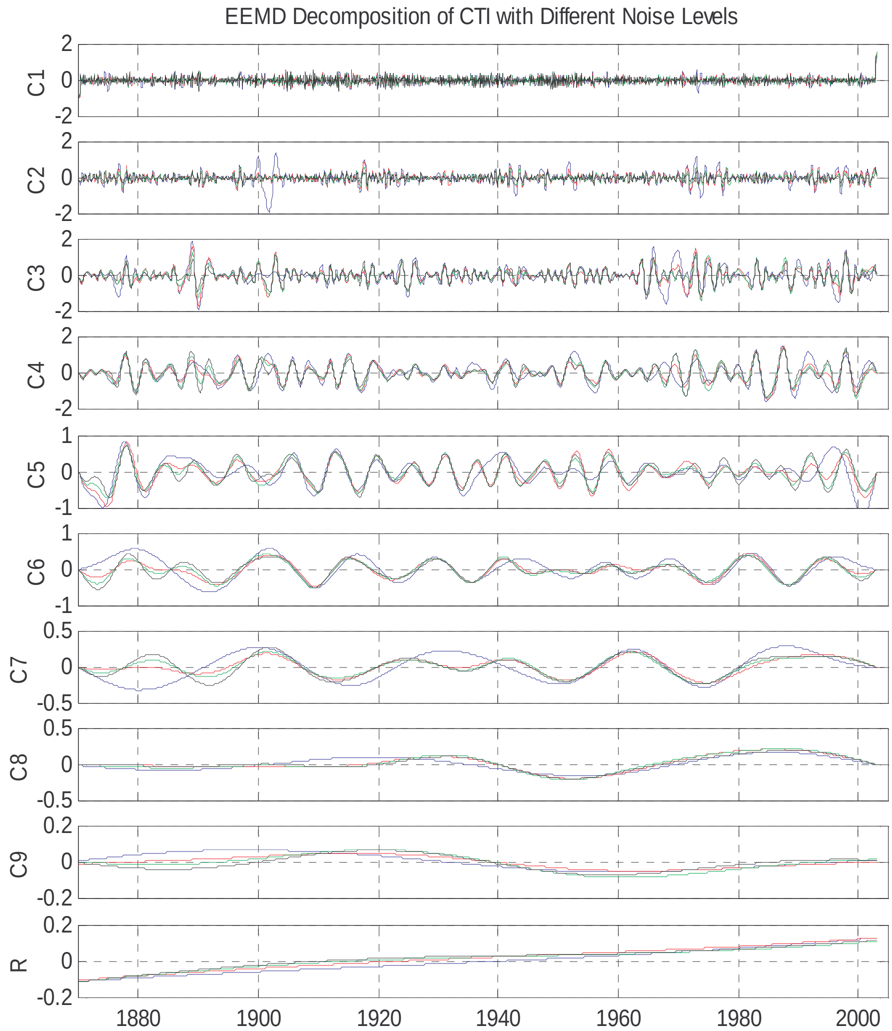

Figure 12: Same as Figure 11, but for CTI.

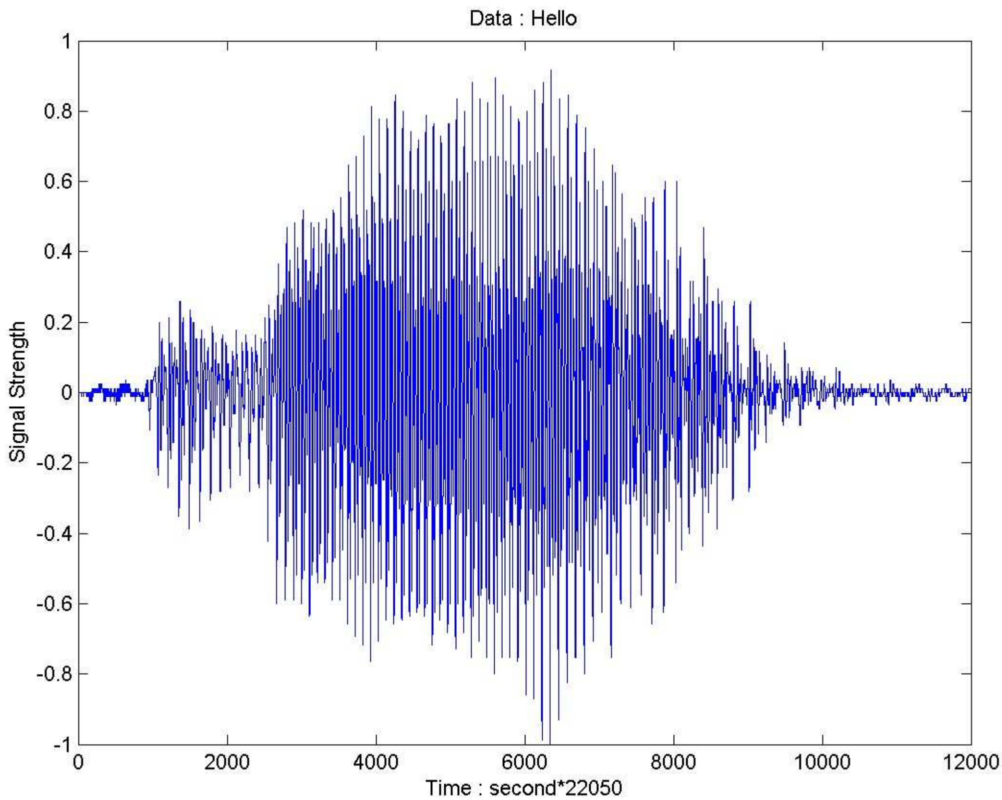

Figure 13: Digitalized sound of the word, ‘Hello’, at 22,050 Hz.

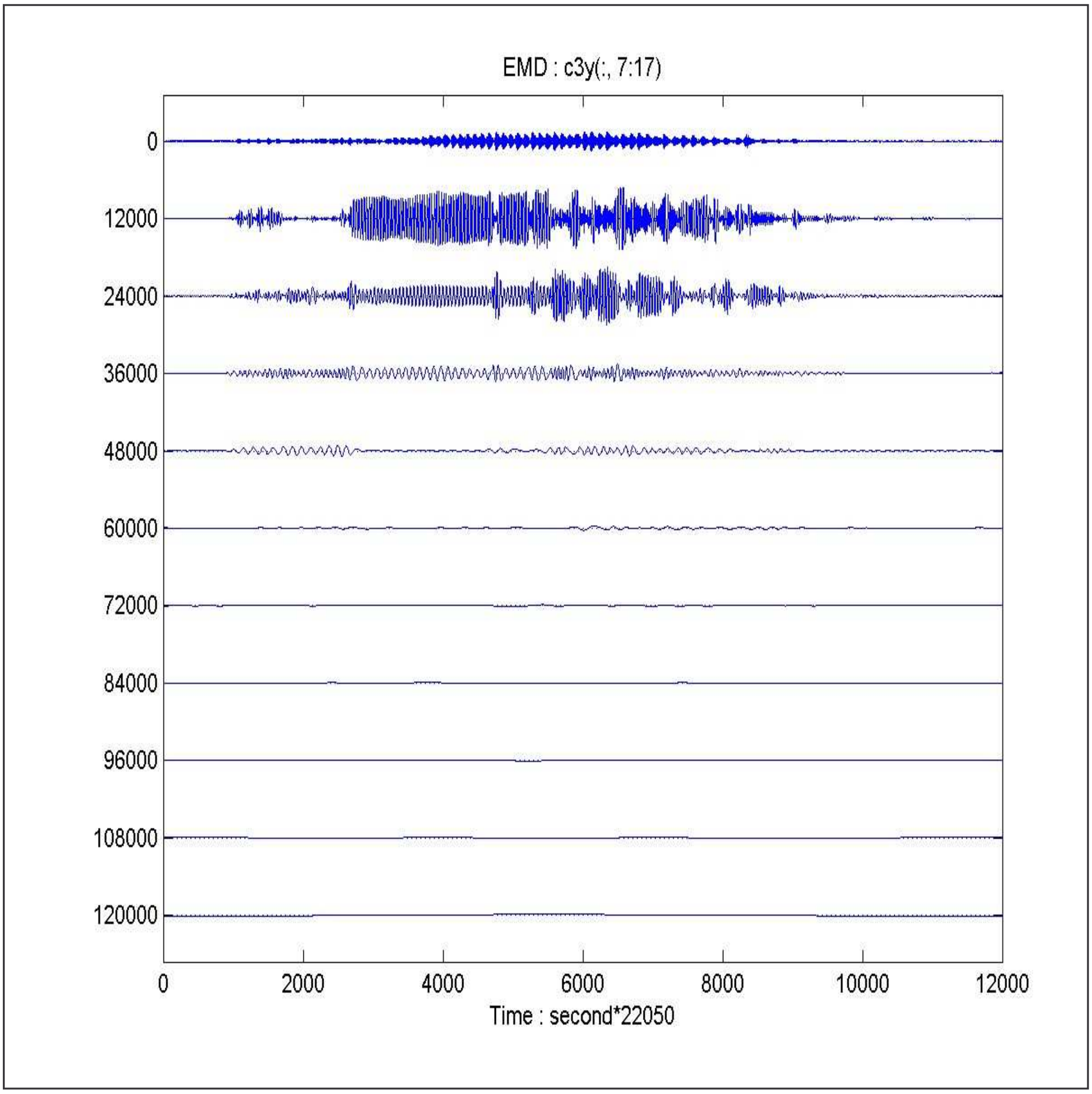

Figure 14: The IMFs (C1 to C11, from the top blue curve to the bottom blue curve, respectively) of digitalized sound "Hello" from the EMD without added noise. The Mode mixing has caused the second and third components to intersperse with sections of data having highly disparate amplitudes and scales.

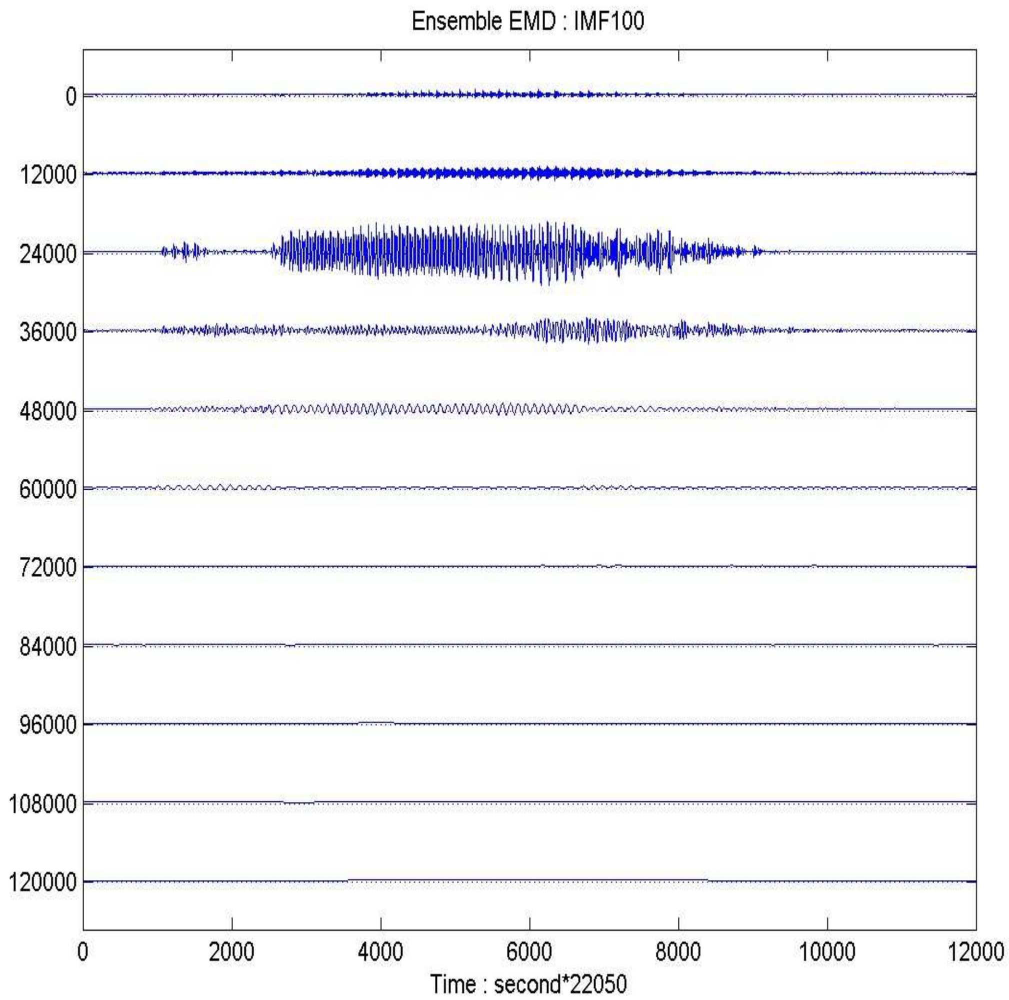

Figure 15: The IMFs (C1 to C11, from the top blue curve to the bottom blue curve, respectively) of digitalized sound "Hello" from the Ensemble EMD with added noise. The Mode mixing has all but disappeared. Each IMF component has consistent amplitudes and scales.

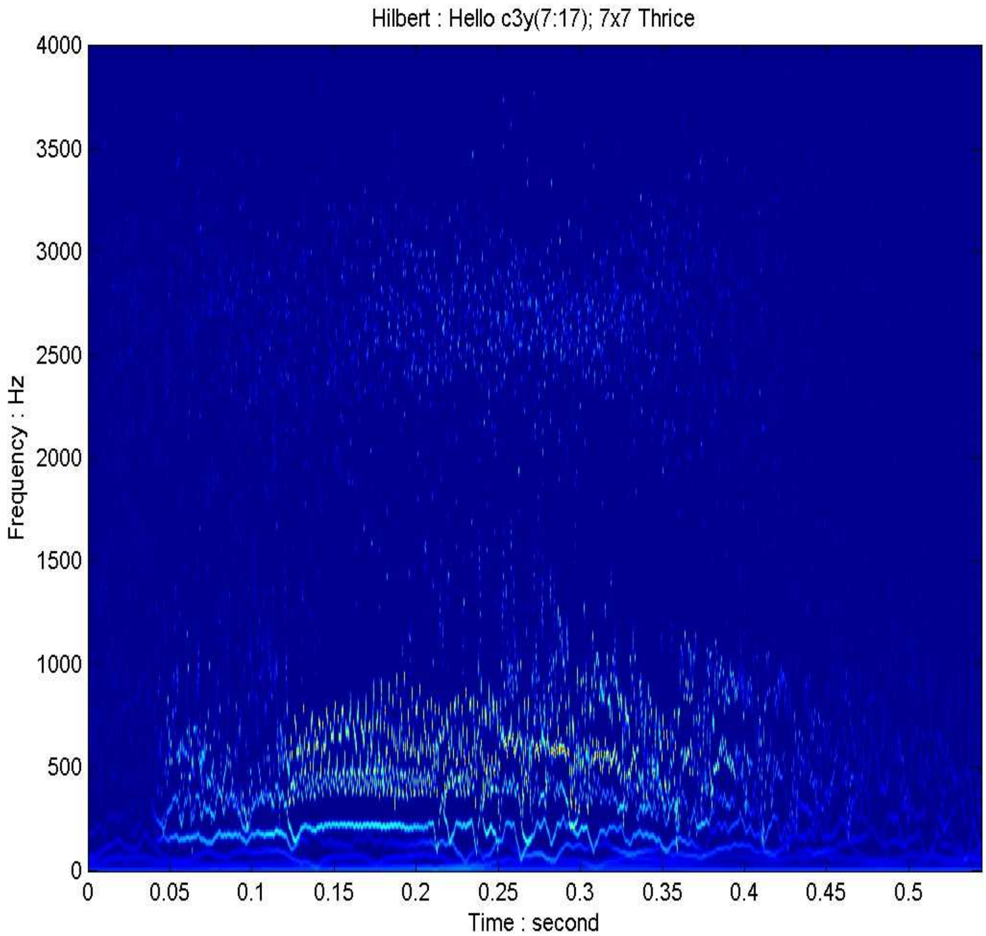

Figure 16: The Hilbert spectrum from the original EMD without added noise. The Mode mixing has caused numerous transition gaps, and rendered the time-frequency traces fragmented.

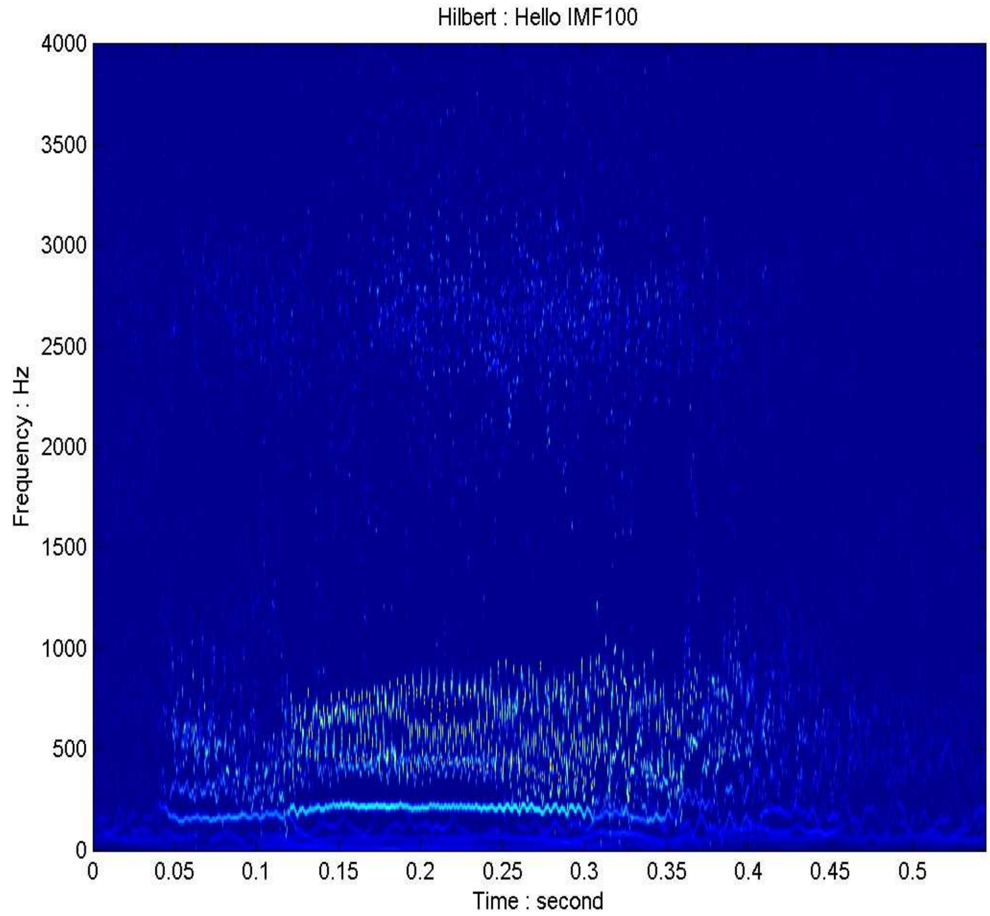

Figure 17: The Hilbert spectrum from the Ensemble EMD with added noise. The Mode mixing has disappeared. There are no transition gaps and all basic frequency traces are continuous in the time-frequency space.