STATUS OF STOCHASTIC DYNAMICS AND SOME RECENT DEVELOPMENTS

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Within the scope of solid mechanics, the problems of stochastic dynamics may be illustrated with a beam-column governed by

(1) $$\frac{\partial^2}{\partial x^2} EI \frac{\partial^2 W}{\partial x^2} + [P + F_1(t)]\frac{\partial^2 W}{\partial x^2} + m\ddot{W} + c\dot{W} = F_2(t)\delta(x - \frac{\ell}{2})$$

In the special case in which EI, P, m, c are constants, and the response $W(x,t)$ can be approximated by one fundamental mode; namely $W(x, t) \approx \phi(x)X(t)$, Eq.(1) can be simplified as

(2) $$\ddot{X} + 2\zeta\omega_0\dot{X} + \omega_0^2\left[1 + \xi_1(t)\right]X = \xi_2(t)$$

or

(3) $$\dot{X}_1 = X_2$$
$$\dot{X}_2 = -2\zeta\omega_0 X_2 - \omega_0^2\left[1 + \xi_1(t)\right]X_1 = \xi_2(t)$$

It is important to note that while equation set (3) has a linear form, the existence of the parametric excitation $\xi_1(t)$ makes the superposition principle not applicable. Systems of this type are known as being quasi-linear.

More generally, the equations of motion may be cast in the form

$$\dot{X}_j = f_j(\boldsymbol{X}, t) + g_{jk}(\boldsymbol{X}, t)\xi_k(t)$$

where X_j are components of vector $\boldsymbol{X}$, $\xi_k(t)$ are stochastic processes, and a repeated subscript implies summation. If the physical properties of a system are assumed to be deterministic, then the functional forms of f_j and g_{jk} are deterministic. Given the probabilistic or statistical properties of the excitations $\xi_k(t)$, several topics of the response will be discussed in the presentation: (1) probabilistic or statistical properties of the system response $\boldsymbol{X}(t)$, (2) motion stability, and (3) probabilistic and statistical properties of the first-passage type failure. Some other contemporary problems will also be discussed.