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NOISE-MODULATED EMPIRICAL MODE DECOMPOSITION

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The empirical mode decomposition (EMD) is the core of the Hilbert–Huang transform (HHT). In HHT, the EMD is responsible for decomposing a signal into intrinsic mode functions (IMFs) for calculating the instantaneous frequency and eventually the Hilbert spectrum. The EMD method as originally proposed, however, has an annoying mode mixing problem caused by the signal intermittency, making the physical interpretation of each IMF component unclear. To resolve this problem, the ensemble EMD (EEMD) was subsequently developed. Unlike the conventional EMD, the EEMD defines the true IMF components as the mean of an ensemble of trials, each consisting of the signal with added white noise of finite, not infinitesimal, amplitude. In this study, we further proposed an extension and alternative to EEMD designated as the noise-modulated EMD (NEMD). NEMD does not eliminate mode but intensify and amplify mixing by suppressing the small amplitude signal but the larger signals would be preserved without waveform deformation. Thus, NEMD may serve as a new adaptive threshold amplitude filtering. The principle, algorithm, simulations, and applications are presented in this paper. Some limitations and additional considerations of using the NEMD are also discussed.

Keywords: Empirical mode decomposition (EMD); ensemble empirical mode decomposition (EEMD); noise-modulated empirical mode decomposition (NEMD).

1. Introduction

The empirical mode decomposition (EMD) is the core of the Hilbert–Huang transform (HHT), which was proposed as an adaptive time–frequency analysis method for nonlinear and nonstationary data [Huang *et al.* (1998; 1999)]. The key innovation associated with EMD is to determine the instantaneous frequency of a signal meaningfully and physically. Using EMD, one can decompose a signal into a set

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of intrinsic mode functions (IMFs) for calculating the instantaneous frequencies through the Hilbert transform and other variations as discussed by Huang *et al.* [2009]. Since its introduction, the method had found wide applications [Huang and Wu, 2008].

It should be noted, however, that the conventional EMD has some drawbacks. One of them is the frequent appearance of the mode mixing, which is defined as a single IMF either consisting of signals of widely disparate scales, or a signal of a similar scale residing in different IMF components. The mode mixing is typically caused by the signal intermittency, which not only causes serious aliasing in the time–frequency distribution, but also makes the physical interpretation of each IMF component difficult and its meaning obscure [Huang *et al.* (1998; 1999)]. To resolve this problem, Huang *et al.* [1999] proposed the intermittence test to alleviate the mode mixing effect. Unfortunately, the intermittence test is based on a subjective selected scale that destroys the adaptive natural of the method. Furthermore, the selection may not be possible if the timescales in the data are not clearly separate, hence indefinable.

To overcome the scale separation problem without a subjective intermittence test, a novel noise-assisted data analysis method, the ensemble EMD (EEMD), was further developed by Wu and Huang [2009]. The concept of the noise-assisted data analysis is similar to the studies by Flandrin *et al.* [2004] and Gledhill [2003], indicating that noise is not useless but actually useful to data analysis by the EMD. The EEMD defines the true IMF components as the mean of an ensemble of trials, each consisting of the signal added to a white noise of finite amplitude. The principle of the EEMD is based on the noise cancellation characteristics. When the signal is added to the uniformly distributed white noise, the noise would populate the whole time–frequency space uniformly with the constituting components of different scales. In this condition, the bits of the signal of different scales are automatically projected onto proper scales of the reference established by the white noise in the background. Different trials may produce noisy results because noise series in each trial is different. Thus, the cancellation of the added noise through the ensemble mean of enough trials is used as the true result.

Recently, we had an interesting finding about the use of the noise-assisted EMD. If we add artificial white noise with infinitesimal amplitude to data and then apply the EMD to the noise added data, the first IMF component obtained will behave like the EMD without noise as discussed by Gledhill [2003]. With the noise level increasing gradually, the smaller amplitude signal in the data that should have appeared in the first IMF component would be suppressed, but the larger amplitude signals would be preserved without waveform deformation. The resulting IMF extracted would have severe mode mixing of course, but judiciously adjusting the noise level actually enables us to achieve an amplitude threshold sensitive filtering. Though the above procedure seems to be similar to that of the EEMD, the spirit is very different: here we use the added noise not to eliminate the mode mixing, but to intensify it. To be effective, the noise amplitude is kept at a much smaller level than that of the finite amplitude noise used in the EEMD. As a result, the added noise amplitude can be increasingly modulated until the small signals we want to suppress are removed. This finding may be viewed as an extension of the noise-assisted data analysis method. We designate it as the noise-modulated EMD (NEMD), for its potential to help handling the thresholding problems, such as signal rejection or amplitude filtering. In Sec. 2, we briefly review the theoretical background, including EMD and EEMD. Section 3 presents the proposed NEMD method, including the principle and algorithm design. Section 4 introduces some simulation examples and practical applications. Section 5 discussed some considerations and limitations of the NEMD.

2. Theoretical Background

2.1. *Empirical mode decomposition*

As mentioned in Introduction, the EMD was developed with the aim of determining the physically meaningful instantaneous frequency of a signal. The details of the EMD can be found in the previous studies [Huang *et al.* (1998; 1999)]. Here we briefly explain the EMD algorithm below.

We first find the local maxima and minima of signal $x(t)$, and use cubic spline interpolation to obtain its upper and lower envelopes. If the mean of these two envelopes is $d_1(t)$, the difference between the signal and $d_1(t)$ is the first component, $h_1(t)$:

$$h_1(t) = x(t) - d_1(t). \tag{1}$$

This is called the sifting process. We have to judge whether $h_1(t)$ is an IMF. Ideally, if the cubic spline interpolation is perfect and there is no gentle hump on the signal slope, $h_1(t)$ should satisfy all IMF requirements. However, in reality imperfect fitting commonly produces overshoots and undershoots that generate new extrema and shift or exaggerate the existing ones. Even if the fitting is perfect, humps may become local extrema after the first round of sifting. The envelope mean may also differ from the true local mean of the signal for nonstationary data, resulting in an asymmetric waveform. Therefore, the sifting process has to be repeated k times until the difference extracted is an IMF. In the second iteration, $h_1(t)$ is treated as the original data in the second sifting process:

$$h_1(t) - d_{11}(t) = h_{11}(t). \tag{2}$$

The sifting process is repeated k times until we find $h_{1k}(t)$, which is an IMF:

$$h_{1(k-1)}(t) - d_{1k}(t) = h_{1k}(t). \tag{3}$$

Then we define

$$c_1(t) = h_{1k}(t) \tag{4}$$

as the first IMF component (i.e. component C1) for the data. Overall, C1 contains the finest and the shortest period component of the signal. Subsequently, we can subtract $c_1(t)$ from the signal:

$$x(t) - c_1(t) = S_1(t). \tag{5}$$

Because residue $S_1(t)$ still contains information about components with longer periods, we treat it as the new original data and apply the same sifting process as described above. This procedure can be repeated for all subsequent $S_j(t)$ values, yielding

$$S_1(t) - c_2(t) = S_2(t), \ldots, S_{n-1}(t) - c_n(t) = S_n(t). \tag{6}$$

Summing Eqs. (5) and (6) finally yields

$$x(t) = \sum_{i=1}^{n} c_i(t) + S_n(t). \tag{7}$$

This indicates that $x(t)$ is decomposed by EMD into n IMFs and a residue $S_n(t)$, which is the signal trend with at most one extremum or a constant.

If one follows the above-described algorithm, the resulting IMF component could have mode mixing effect [Huang *et al.* (1999)], which is defined as any IMF consisting of oscillations of dramatically disparate scales. The mode mixing is often caused by the intermittency of the driving mechanisms, and it would make the physical meaning of each IMF component unclear.

2.2. *Ensemble empirical mode decomposition*

The EEMD has been proposed to establish EMD as a firm dyadic filter bank. At the same time, it also resolves the mode mixing problem. The principle and the details of EEMD are given in the paper by Wu and Huang [2009]. The procedure of the EEMD is as follows: (1) add a white noise series to the data. The noise amplitude is typically set to be 0.1 times the standard deviation of the original data or higher; (2) use the EMD to decompose the noise-added data into IMFs; (3) repeat Steps 1 and 2 again and again, but with different white noise series each time; and (4) the ensemble means of the corresponding IMFs of the decompositions are used as the final result. The effects of the decomposition using the EEMD are that the added white noise series cancel each other in the final mean of the corresponding IMFs. The mean temporal scale of the IMFs would stay within the natural dyadic filter windows; therefore, EEMD can significantly reduce the chance of the mode mixing and preserve the dyadic property [Wu and Huang (2009)].

3. Noise-Modulated Empirical Mode Decomposition

3.1. *Algorithm*

The NEMD can be treated as an extension and variation of the EEMD. The concept of the NEMD came from our pilot studies and observations described below. When we carried out EEMD of a signal using an infinitesimal noise level instead of using finite noise amplitude, the first IMF mode (i.e. C1 component) would behave like the IMF results without noise as discussed by Gledhill [2003]. It is, however, interesting to note that if we gradually increase the noise amplitude, the smaller amplitude signals in the data would be suppressed, but the larger signals would be preserved without any deformation of the waveform. Consequently, the mode mixing in the resulting IMF is actually intensified and amplified. Interestingly, although the NEMD is very similar to the EEMD, with infinitesimal level noise added, it can perform a different function in data analysis as an effective amplitude threshold filter. The difference between the NEMD and EEMD is just that the amplitude of the added white noise for the NEMD is much smaller than that for the EEMD. Actually, the added noise would be considered as ineffective in EEMD. Yet, the added low-intensity noise amplitude can be modulated incrementally until the small signals we want to suppress are removed. Therefore, the purpose of using NEMD is very different from EEMD: it is used not to eliminate mode mixing, but to amplify it.

In order to implement the NEMD, we designed the algorithm according to the illustration of Fig. 1 in the following steps: (1) add artificial white noise to the

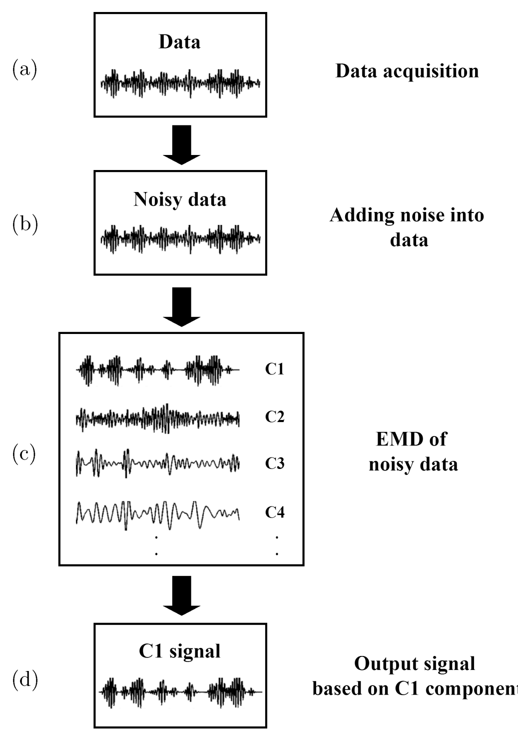


Fig. 1. Flow chart for the algorithm of the NEMD: (a) the data, (b) the noisy data obtained by adding white noise to the signal in (a), (c) EMD of the noisy data, and (d) taking component C1 as the result of the NEMD.

data, and the noise level should be much smaller than the data; (2) apply EMD to the noise added data, and take the C1 IMF component as the output signal; (3) observe the C1 signal, and identify whether the performance of the signal rejection is satisfying for needs; (4) if not satisfying, modulate the added noise level to gradually increase and repeat Steps 1 and 2 until the small signals we want to remove are suppressed in the C1 component. Undoubtedly, increasing the noise level added into the data would result in more signals rejected (i.e. like using a larger threshold value).

3.2. *Basic principle*

Ideally, white noise has an infinite bandwidth and hence is likely to contain fluctuations at higher frequencies than those in the data; therefore, the added noise has the ability to endow the waveform of the data with additional local extrema. Adding an appropriate level of white noise to the data will change the probability on the detections of local extrema for large and small amplitude waveforms in the signals. Waveform for the large amplitude oscillations would have a relative steeper slope on the waveform; therefore, the extrema of the added noise would not show up. Whereas those for small amplitude waveforms, the extrema from the noise would show up and be altered by the added noise and even replaced by the extrema of the added noise. In this situation, the first IMF component obtained from EMD of the noise added signals just shows larger amplitude waveforms, whereas the part corresponding to small amplitude waveforms would be replaced with the noise-scale baseline. Thus, the IMF extracted would consist of the large amplitude waveforms mixed with the noise level high-frequency fluctuations portion. Though the resulting IMF contains severe mode mixing, the small amplitude waveforms are effectively rejected. Thus, we have in this low noise intensity EEMD, or NEMD, an effective amplitude threshold filter.

4. Simulations and Applications

4.1. *Simulations*

Having presented the basic principles and implementation steps, we will use computer simulations to demonstrate and validate the algorithm of the NEMD. The ultrasonic backscattering model was used as the simulation method for producing the simulated signals. The reason we used ultrasound data as the test signal is that the rejection of small signal is necessary in medical ultrasound image for noise reduction and contrast enhancement as discussed in Tsui *et al.* [2008; 2009]. The details for the simulation method and the results on the test of the NEMD algorithm are described as follows, with more details given in Tsui *et al.* [2008].

The two-dimensional model of ultrasound signal was used to produce ultrasonic radio-frequency (RF) data to verify the algorithm of the NEMD, as given by

$$RF = A\{[H \otimes Z] \cdot e^{-\alpha y} + N\}, \tag{8}$$

where A denotes the gain factor of the echo-receiving system, H represents the transducer transfer function, and Z is the spatial distribution function of the scatterers. Note that Z is essentially a two-dimensional matrix with randomly positioned delta functions weighted by the backscattering coefficient that describes the spatial arrangement of the scatterers in the medium. The exponential term in Eq. (8) accounts for the attenuation effect, with an attenuation coefficient of α, and N is the signal-independent white noise.

In the following example, we used an ideally simulated RF data as an example, in which the attenuation and noise effect were not involved in simulations. At first, an incident wave with a central frequency of 7.5 MHz and a bandwidth of 80% was used to generate the ultrasound RF data. The sampling rate was 100 MHz. Subsequently, we added white noise to the clean RF data to produce noise-added RF signals with different SNRs. Specifying the power of the clean RF data as 0 dB, we adjust the SNR of the noise-added RF signal from 40 to 0 dB using the "awgn" function in MATLAB software.

Figures 2–5 show the original RF data, noise-added data, and C1 IMFs obtained from NEMD of the noise-added RF signals with SNRs ranging from 40 to 5 dB. For an SNR of 40 dB, we found that the waveform of the component C1 was nearly identical to that of the original ultrasonic signal, as shown in Fig. 2. This is because

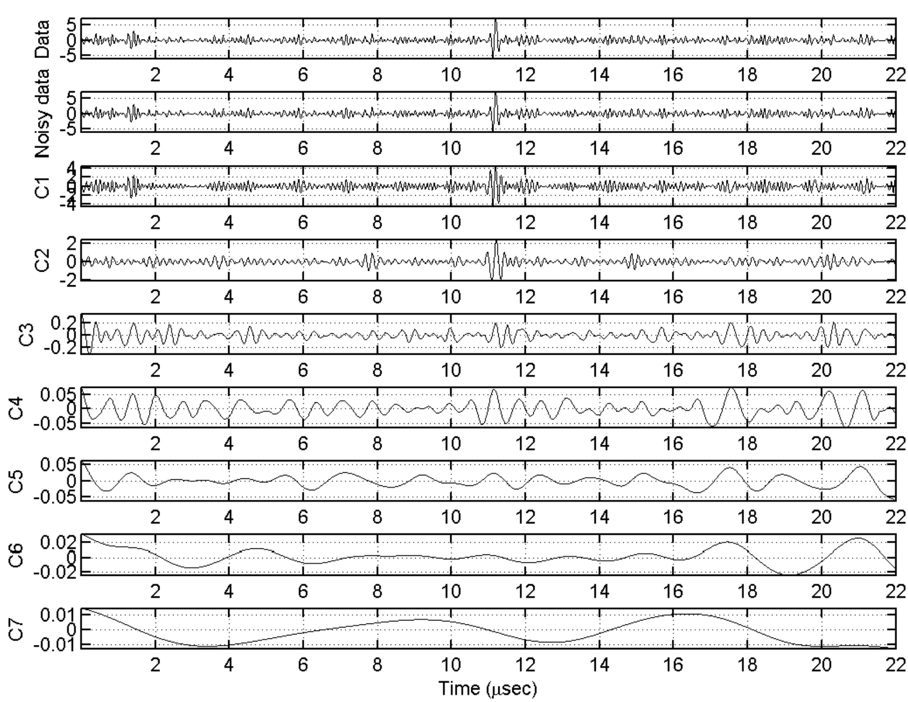


Fig. 2. The original data, noise-added data, and IMFs obtained from NEMD of the noisy data for the SNR of 40 dB.

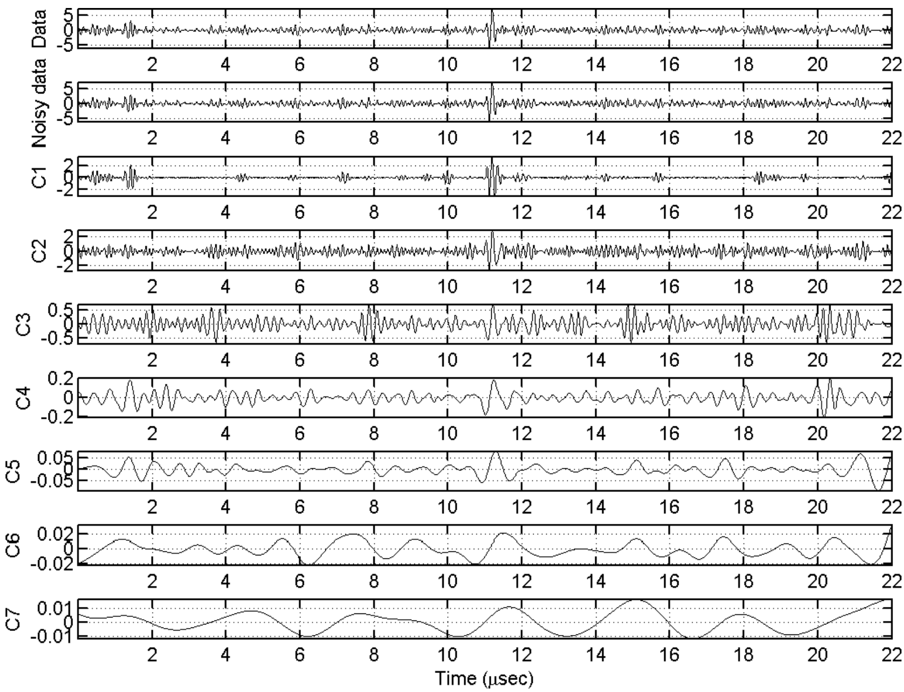


Fig. 3. The original data, noisy data, and IMFs obtained from NEMD of the noise-added data for the SNR of 20 dB.

the power of the added noise was much lower than that of the ultrasound signal when the SNR is 40 dB, and hence did not impact the local characteristics of the ultrasound signal. This means that the local extrema acquired from the noise-added RF signal are entirely due to the ultrasound RF signal itself, and hence the C1 signal behaves like the original RF data but with a more symmetric waveform. On the other hand, we further found that different oscillatory modes of the RF signal were evident in IMF components C2–C5. The oscillatory frequency decreases with increasing IMF index. Some of the final components (e.g. C6–C7) are empirically treated as the signal trends.

For an SNR of 20 dB (Fig. 3), the larger echoes and some background small signals were extracted from the noise-added RF data to form the C1 component. Comparison of Figs. 2 and 3 indicates that the noise level affects the results of the NEMD, which is further confirmed by the C1 IMF results for the noisy signal with an SNR of 15 dB as shown in Fig. 4. Obviously, adding higher intensity white noise to the ultrasonic signal results in only the extracted larger signals forming the component C1. Specially, the waveform and the corresponding locations of the preserved signals do not have any significant change. When much higher intensity noise has been added, most local extrema acquired from the noise-added data would come from the noise-induced fluctuations, and thus the C1 component simply describes

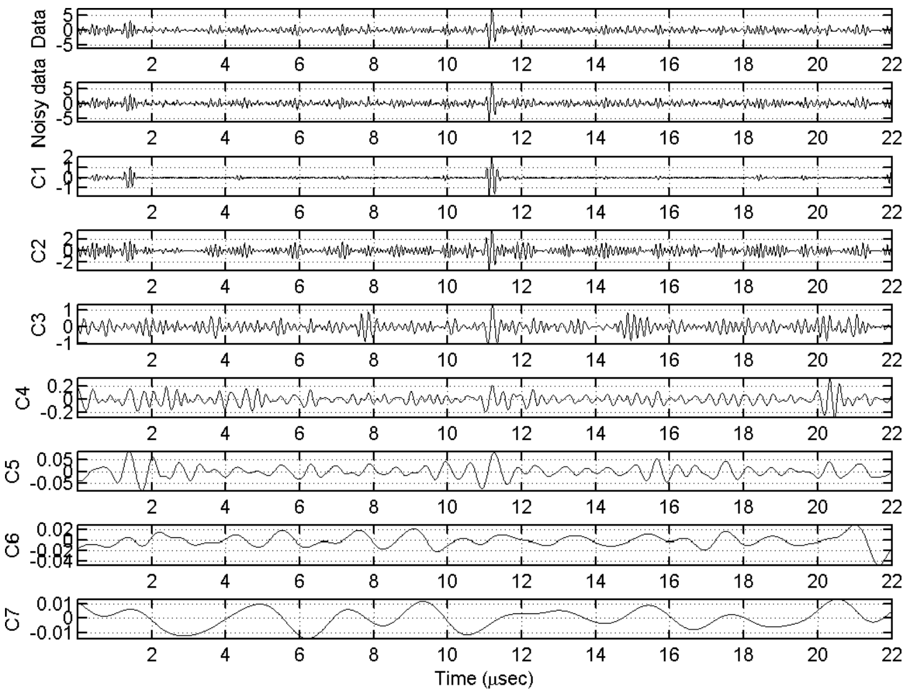


Fig. 4. The original data, noisy data, and IMFs obtained from NEMD of the noise-added data for the SNR of 15 dB.

the noise characteristics to mean that most echo signals are rejected, as shown in Fig. 5 for the SNR of 5 dB.

The above simulation demonstrated that the NEMD indeed has good ability to suppress the small amplitude signals without destroying the waveform features of the preserved large signals. Compared to the conventional threshold filter based on one specific threshold value, the NEMD may be treated as an adaptive amplitude threshold filter.

4.2. *Applications*

Here we introduce practical applications of the NEMD in an ultrasound image. We carried out image measurements on the cyst in a breast phantom to acquire the ultrasonic RF signals. The tissue-mimicking breast phantom was constructed by Professor Ernest L. Madsen, Department of Medical Physics, University of Wisconsin-Madison. The ultrasound RF signals from the cyst were acquired using a commercial ultrasound imaging system (Model 2000, Terason, Burlington, MA, USA), with the raw RF data digitized at a sampling rate of 30 MHz. The applied probe is a wideband linear array with a central frequency of 7.5 MHz and 128 elements (Model 10L5, Terason). The pulse length and the bandwidth were 0.5 μs and 60%, respectively. The lateral beamwidth is about 0.5 mm, and the focal length is

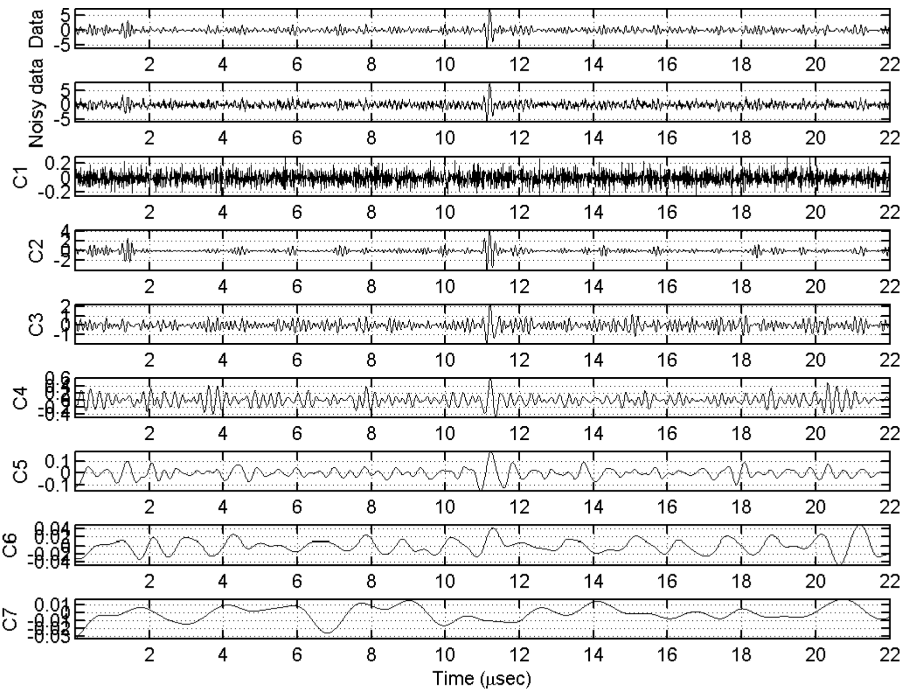
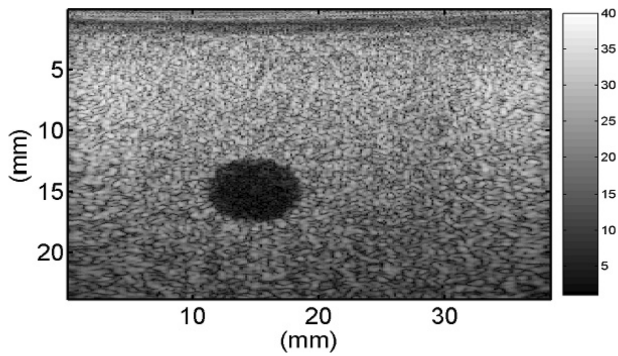


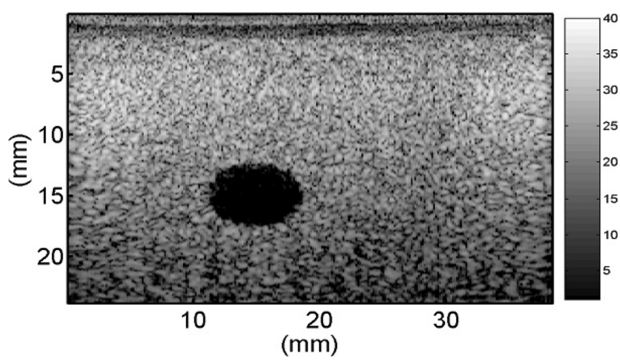
Fig. 5. The original data, noisy data, and IMFs obtained from NEMD of the noise-added data for the SNR of 5 dB.

adjustable. The signal interpolation was further performed to increase the sampling rate to 100 MHz to be the same as that of simulation. The B-mode image was then formed using the log-compressed envelope image with a dynamic range of 40 dB.

The typical ultrasound B-mode image of the cyst is shown in Fig. 6(a). It can be expected that the small echoes due to noise effect or ultrasonic backscattering from very small and few scatterers in the cyst would reduce the contrast-to-noise ratio to affect the ability of the B-scan to detect the cyst target (i.e. contrast resolution). This is because the cyst is theoretically an anechoic area, and therefore there should not be any returned signals. Now in order to reduce the effect of small signal on the image quality of the cyst, we used the NEMD to reject the small echoes in the cyst. For each scan-line, we added white noise by adjusting the signal SNR to be 25 dB and then applied the EMD of the noise added signal to form the B-mode images with the same dynamic range, as shown in Fig. 6(b). Evidently, the small signals are depressed so that the cyst in the B-mode image becomes darker, improving the image contrast. More importantly, the NEMD process does not cause any deformation of the waveform features for the preserved significant echoes, as indicated by Fig. 7. Obviously, the NEMD performs better than the conventional threshold technique, for NEMD could reject the small backscattering echoes without waveform distortions.



(a)



(b)

Fig. 6. B-mode images of the cyst acquired from the breast phantom (a) before and (b) after applying the NEMD with an SNR of 25 dB.

5. Discussion and Conclusions

This study proposed the NEMD, which is a new noise-assisted data analysis method based on the conventional EMD and the concept of the improved EEMD. We used the simulations and experiments to demonstrate the feasibility and practical applications of the NEMD for improving the image contrast. The main function of the NEMD is to suppress small signals in the data by modulating the noise level with infinitesimal amplitude appropriately. The simulation results show that the NEMD can reject small signals but, at the same time, preserve the waveform of large signal without distortions. The examples further demonstrated that the NEMD can indeed have practical applications in the field of biomedical ultrasound. Evidently, the NEMD can be treated as an adaptive amplitude threshold filter for applications of signal thresholding process.

However, there may be some considerations and limitations when the NEMD is used in practice, as discussed below. (1) Different systems would produce different dynamic ranges and different degrees of noise interference for the data measured

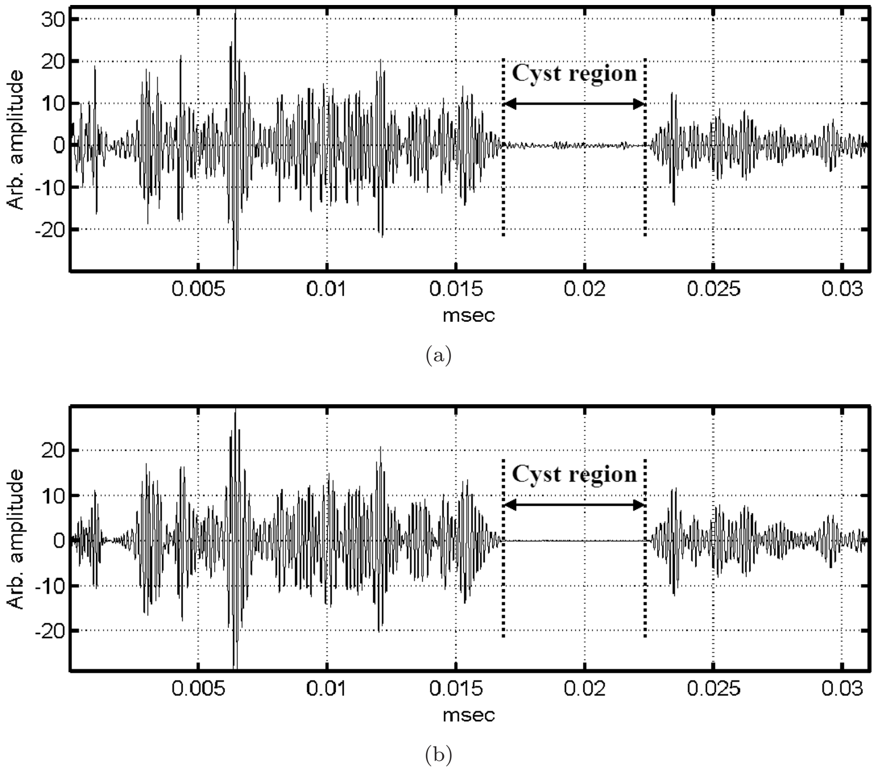


Fig. 7. Ultrasound RF signals acquired from the cyst in the breast phantom (a) before and (b) after applying the NEMD with an SNR of 25 dB.

from the same target. It means that there is no optimal level for the artificial noise added into the data to perform the NEMD. The noise level needed for the NEMD may have to be redetermined or corrected for each specific application when using different systems. In other words, the applications will have to go through a training stage to obtain the optimum operation mode for each specific case. (2) The NEMD may not work when the original noise interference on the measured data is too strong. Too much noise reduces the signal quality of the data to a near uniformly noisy field, which would make the output of the NEMD based on the first IMF component be just a noise-based signal. (3) It has been shown that EMD of a signal would be influenced by the sampling rate used to digitalize the analog data [Rilling and Flandrin (2006); (2009)]. According to the Nyquist theorem, a half of the sampling rate will determine the maximum bandwidth of the added white noise. Thus, the bandwidth of artificial white noise used for the NEMD is dependent on the sampling rate. In other words, the sampling rate certainly affects the NEMD. The above issues need to be further investigated before the NEMD is used as a reliable signal processing tool.

The NEMD method introduced here had been used in ultrasonic applications in the biomedical field with great success [Tsui *et al.* (2008; 2009)]. The same method could easily be adapted to other fields such as radar image and general image analysis and processing. It is with the general applications in mind that we decided to introduce it here as a new data analysis tool.

Acknowledgments

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